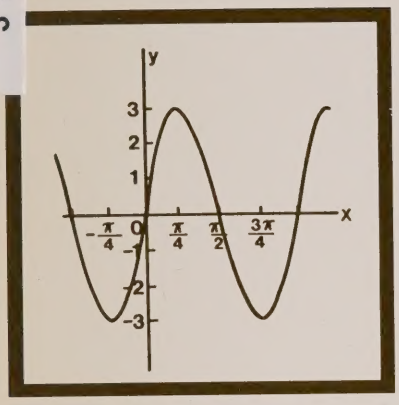
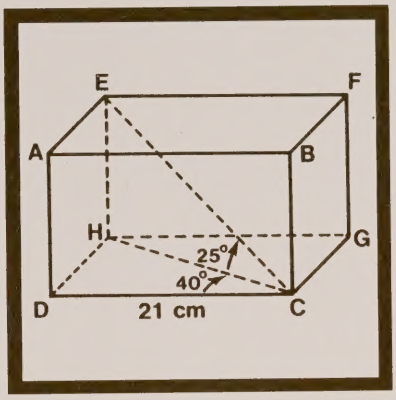
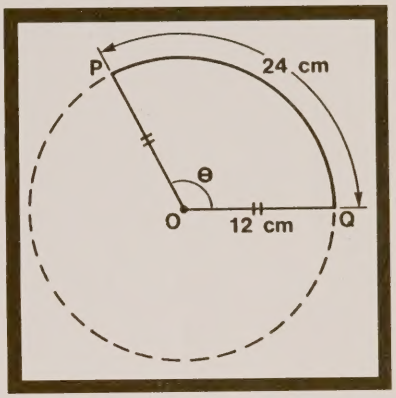
MATHEMATICS

Grade 12
(Advanced Level)

A REPORT FOR EDUCATORS







Ministry of Education
Ontario

PROVINCIAL REVIEWS

MATHEMATICS

Mathematics
Grade 12 (Advanced Level)
A Report for Educators

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This review would not have been possible without the dedication shown by the many individuals who worked on various committees. Members of the main review committees are identified in Appendix G. We also want to thank the Lakehead Board of Education and the Metropolitan Separate School Board for their co-operation in making it possible to have photographs taken in their classrooms.

Executive Summary

Overview

The provincial review of mathematics for Grade 12 (advanced level) is part of an ongoing program of provincial reviews instituted by the Ministry of Education. These reviews are the primary means by which the ministry carries out its commitment to monitor education in the Province of Ontario and inform the public about the performance of the educational system.

In order to obtain a direct indication of how well Grade 12 (advanced level) students have achieved the goals of the mathematics program, the Grade 12 (advanced level) students in 99 English-language schools answered multiple-choice and open-ended questions in geometry, relations and functions, algebraic operations, and applying problem-solving strategies to process problems.

Questionnaires answered by principals, teachers, and students provided information on teaching practices and student attitudes.

Student Performance on Mathematics Tasks

Grade 12 (advanced level) students achieved at levels that were judged to be satisfactory or better in half of the topics in the course; achievement was rated as *strong* in one topic, *satisfactory* in five topics, *marginal* in five topics, and *weak* in one topic.

Achievement was *strong* in the topic of similar figures; it was *satisfactory* in congruence and parallelism, the circle, polynomials and equations, radicals, and process problems. The following were identified as areas of weakness: graphs of functions and relations, trigonometric functions, applications of trigonometric functions, exponential and logarithmic functions, graphs of second-degree relations, and absolute value.

Student Attitudes

Most students expressed positive attitudes towards mathematics. The majority thought that mathematics was important for employment, that it was of equal importance for boys and girls, and that it was helpful to them outside of school. Fifty-one per cent of students felt that they were good in mathematics, and 33 per cent felt that mathematics was

difficult. Sixty-one per cent of students liked mathematics. When given a difficult problem, most students attempted a variety of approaches to reach a solution; few students gave up before reaching a solution.

Classroom Activities and Classroom Practices

The most commonly used methods of instruction were chalkboard or projector presentations to the whole class and having students work individually on assignments. Teacher demonstrations with concrete materials were used for an average of 8 per cent of the class time.

Calculators were often used in most classrooms, but approximately half of the teachers reported that students did not have access to computers. When computers were available, teachers most frequently encouraged students to use them to practise new concepts, to learn new concepts and skills, and as an aid in doing projects.

Most students reported that they frequently worked on mathematics alone and that their teacher frequently taught mathematics to the whole class. More than half of the students never did projects in mathematics, and most students never used a computer in mathematics class.

Fifty-six per cent of students had computers at home, but only 4 per cent used them for mathematics school work.

Seventy-three per cent of students did one or more hours of mathematics homework each week, 22 per cent did less than one hour, and 5 per cent did no mathematics homework.

Teachers reported that students had the opportunity to learn the course content required to answer most of the questions in the Student Assessment Booklets. Opportunity to learn was highest for algebraic operations; it was also high for relations and functions, but somewhat lower for geometry.

It is hoped that this report will encourage discussion about mathematics in Ontario schools. Ontario educators should now take steps to:

- maintain the strengths that were identified by this review of the Grade 12 (advanced level) mathematics program;
- recommend and implement procedures that will lead to improvement in areas where weaknesses were identified.

1. Purpose of the Review

The provincial review of mathematics for Grade 12 (advanced level) is part of an ongoing program of provincial reviews conducted by the Ministry of Education. These reviews are the primary means by which the ministry carries out its commitment to monitor education in Ontario and inform the public about the performance of the educational system. They are based on Ministry of Education curriculum guidelines and provide information about program implementation and student achievement. The main purposes of the reviews are to:

- determine the effectiveness of curricula and the level of student achievement at the school, board, and provincial levels;
- provide Ontario educators with information that may be used to make decisions for modifying and improving programs and improving levels of student achievement;
- establish baseline data on both program effectiveness and student achievement against which information generated by future reviews can be compared in order to determine progress;
- provide instruments and procedures that can be used in the future by school boards.

The provincial review of mathematics for Grade 12 (advanced level) was carried out for two main reasons: first, to generate a profile of the nature of mathematics instruction in the Grade 12 (advanced level) course in Ontario; and, second, to determine how well students enrolled in this course are achieving the goals of the mathematics program.

The profile of mathematics instruction describes some of the characteristics of students enrolled in mathematics at the advanced level in Grade 12, their teachers, and the mathematics activities that occur in their classes. It also compares students' achievement on specific tasks with the performance expectations of parents, trustees, and educators.

Relating the profile to our current knowledge of the nature of mathematics and to the Ministry of Education's policy and guidelines on mathematics instruction will assist in identifying the most appropriate directions for mathematics instruction in Grade 12 (advanced level) courses to follow during the 1990s. The identification of these directions can be facilitated by an analysis of the responses to the Principal Questionnaire, the Teacher Questionnaire, the Student Questionnaire, and the Student Assessment Booklets that were administered as part of this review.

This report presents in detail the results of the provincial review of mathematics for Grade 12 (advanced level) conducted in the spring of 1990. A summary document, *Mathematics, Grade 12 (Advanced Level): A Report Card for Ontario,* has also been prepared. Parallel reports based on data from French-language schools are also available.

2. Mathematics in Ontario Schools

Mathematics is an essential skill for literate citizens because it is required in all areas of life, including the home and the marketplace as well as the school. It includes the study of patterns, the creation and application of abstract systems, and the use of logical arguments. For these reasons, the study of mathematics is an integral part of the education of all students up to Grade 10. It should help them appreciate the contribution that mathematics has made to the development of civilization, as well as prepare them for postsecondary education and for future careers. Beyond Grade 10, mathematics becomes more specialized. Grade 12 (advanced level) mathematics is intended mainly for students who wish to study mathematics at college or university.

As part of its province-wide responsibility, the Ministry of Education, Ontario, establishes goals, policies, and specific expectations through its curriculum guidelines. The provincial curriculum policy for Ontario programs in the Intermediate and Senior Divisions is outlined in *Ontario Schools, Intermediate and Senior Divisions (Grades 7-12/OACs): Program and Diploma Requirements, 1989,* rev. ed. *(OSIS).* The specific expectations for Grade 12 (advanced level) mathematics are provided in the curriculum guideline *Mathematics, Intermediate and Senior Divisions, 1985.*

While the ministry sets goals, policies, and specific expectations, it is the responsibility of local school boards to formulate programs that are within the rationale of the provincial policy and, at the same time, reflect local needs and priorities. Teachers, working both individually and as a group under the leadership of the department head, have the task of planning and delivering classroom programs that are specifically adapted to the students for whom they are responsible. This task includes the selection of appropriate teaching strategies, resources, and learning activities.

In the past, research has made significant contributions to the development and implementation of mathematics programs in Ontario's schools. It is expected that the results of this provincial review will be similarly beneficial.

3. Description of the Measures and Samples for the Provincial Review

3.1 The Instruments

This provincial review used the following instruments to create a multifaceted profile of the mathematics experiences of students enrolled in mathematics at the advanced level in Grade 12: Principal Questionnaire, Teacher Questionnaire, Student Questionnaire, and Student Assessment Booklets. All of these instruments are included in a separate publication entitled *Mathematics, Grade 12 (Advanced Level): Review Instruments*. As well, planning documents were submitted to the ministry for review.

- The **Principal Questionnaire** was used to create a profile of the school environment and curriculum-management practices in the school. The questionnaire covered the following areas: the number of students enrolled in and the time allocated to mathematics instruction, the community served, the procedure for reviewing mathematics programs, and the extent of professional activities in mathematics education.

- The **Teacher Questionnaire** was used to collect information on teaching experience, personal and professional backgrounds of teachers, resources, teaching methods, and evaluation procedures.

- The **Opportunity-to-Learn Form** was used to collect information as to whether or not students had been taught the concepts and skills necessary to answer the questions in the Student Assessment Booklets.

- The **Student Questionnaire** provided information about the background and attitudes of students. Students answered questions about themselves and some of their activities outside of school, their attitudes towards school and mathematics, and what they did during mathematics class.

- **Student Assessment Booklets** were used to measure student achievement in mathematics. Each of four different booklets contained questions related to all topics of the mathematics program. Most of these were multiple-choice questions, but three questions in each booklet required a written response. Some of these latter questions required students to supply a numerical response, some required students to complete a graph, some required students to write the algebraic solution for an equation, and others required that students undertake the full solution of a problem or a proof. Students were allowed to use calculators in completing the questions in these booklets. In addition, four process problems were administered to measure problem-solving skills.

The following objectives of the course were assessed in the provincial review:

- **Geometry**

  — *Congruence and Parallelism*

    - identifying as axioms properties such as the sum of the angles of a triangle; angle properties related to parallel lines, intersecting lines, and isosceles triangles; sufficiency conditions for congruent triangles; and basic properties of reflection, rotation, and translation
    - analysing and proving simple deductions and theorems
    - solving problems involving numerical applications
    - identifying the features of indirect proof

  — *Similar Figures*

    - defining similar figures
    - identifying the sufficiency conditions for similar triangles
    - solving numerical problems and proving simple deductions based on the sufficiency conditions for similar triangles
    - identifying and proving the relationship between linear dimensions and the area of similar figures

— *The Circle*

- defining terms
- proving angle, chord, tangent, and secant properties associated with a circle and using these properties in numerical problems
- defining the radian measure of angles and converting angle measures from radians to degrees and from degrees to radians
- solving numerical problems involving circumference, area, length of arc, and the area of sector of a circle

▪ Relations and Functions

— *Graphs of Functions and Relations*

- defining function, inverse of a function, and inverse function of a function
- identifying the relationship between the graph of $y = f(x)$ and the graphs of

i) $y = kf(x)$
ii) $y = f(x) + b$
iii) $y = f(x - a)$
iv) $y = f(mx)$
v) $y = kf\,(m(x - a))$
vi) $y = f^{-1}(x)$

- identifying the relationship between the graphs of $y = f(x)$ and $y = g(x)$, and the graphs of $y = f(x) + g(x)$ and $y = f(x) - g(x)$
- defining the $f(g(x))$ and determining the composite of two given functions

— *Trigonometric Functions*

- defining the trigonometric functions $y = \sin x$, $y = \cos x$, $y = \tan x$, $y = \csc x$, $y = \sec x$, and $y = \cot x$; sketching their graphs and identifying features such as range, periodicity, domain, and amplitude
- relating the graphs of $y = \sin x$ and $y = \cos x$ to the graphs of

i) $y = k \sin x$, $y = k \cos x$
ii) $y = \sin mx$, $y = \cos mx$
iii) $y = \sin (x - a)$, $y = \cos (x - a)$
iv) $y = k \sin m(x - a)$, $y = k \cos m(x - a)$
v) $y = k \sin m(x - a) + b$, $y = k \cos m(x - a) + b$

- identifying the amplitude, periodicity, and phase shift for $y = k \sin m(x - a)$ and $y = k \cos m(x - a)$

— *Applications of Trigonometric Functions*

- relating the definitions of the trigonometric functions to right triangles; solving problems involving right triangles
- proving simple identities involving trigonometric functions and the quotient, reciprocal, and Pythagorean relations
- proving the sine and cosine laws; solving problems involving oblique triangles
- solving trigonometric equations within a specified domain

— *Exponential and Logarithmic Functions*

- defining a^x where x is a rational number
- drawing graphs of $y = a^x$
- defining the logarithmic function $y = \log_a x$; identifying the domain of the function
- converting equations from exponential to logarithmic form and from logarithmic to exponential form, such as $y = a^x$ to $y = \log_a x$
- proving the laws of logarithms
- solving problems involving exponential and logarithmic functions
- determining the term of an investment or loan at compound interest; determining the rate of interest, given the principal, the amount, and the time

— *Graphs of Second-Degree Relations*

- sketching and identifying graphs of functions defined by $y = ax^2$; identifying such a graph as a parabola
- identifying the transformation mapping the graph of a circle defined by $x^2 + y^2 = r^2$ to a graph defined by $(x - a)^2 + (y - b)^2 = r^2$
- identifying the transformation mapping the graph of a circle defined by $x^2 + y^2 = 1$ to a graph defined by $\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1$; identifying such a graph as an ellipse
- defining and identifying terms such as major axis, minor axis, and vertices for an ellipse
- sketching the graph of $x^2 - y^2 = 1$; identifying the graph as a rectangular hyperbola

- identifying the transformation mapping the graph defined by $x^2 - y^2 = 1$ to a graph defined by $\left(\frac{x}{a}\right)^2 - \left(\frac{y}{b}\right)^2 = 1$; identifying such a graph as a hyperbola

- **Algebraic Operations**

  — *Polynomials and Equations*

  - factoring polynomials of the form $ax^2 + bx + c$, $a^2x^2 - b^2y^2$, $a^2x^2 + 2abxy - b^2y^2$; factoring by grouping
  - dividing a polynomial by a binomial
  - stating and using the remainder theorem and the factor theorem
  - factoring the sum and difference of cubes
  - solving quadratic equations by factoring and formula
  - solving polynomial equations of degree n where $n > 2$

  — *Absolute Value*

  - defining absolute value
  - solving equations and inequalities involving one variable in which absolute value appears once

  — *Radicals*

  - interpreting and evaluating powers with integral bases and rational exponents
  - defining $\sqrt{x^2}$ and $\sqrt[3]{x^3}$ as principal roots

The following three categories were used to classify questions according to the level of thought required:

- **recognition/recall and comprehension:** the recognition or recall of a definition, concept, formula, or algorithm; or the use of a concept, formula, or algorithm in a routine way requiring little analysis
- **application:** the use of a concept, formula, or algorithm in a problem requiring some analysis
- **analysis/proof:** the solving of a routine problem which requires significant analysis and/or a proof, or the solving of a non-routine problem which requires the choice of a strategy in addition to analysis

Grade 12 (advanced level) mathematics is designed for students who have an intrinsic interest in mathematics and the ability to work with abstract ideas. Students are expected to use appropriate mathematical symbolism in a precise, unambiguous, and concise way. They should develop and confirm generalizations.

3.2 Development and Administration of Instruments

The assessment booklets and questionnaires used in the provincial review of mathematics for Grade 12 (advanced level) were developed by a team of educational experts in the subject of mathematics who collaborated in selecting the areas to be examined and composing the individual questions. Once the booklets were completed, they were presented to an independent panel of consultants, who evaluated their overall validity; that is, the consultants determined whether the questions evaluated students' skills in mathematics at the appropriate level – Grade 12 (advanced level) – in the designated areas. Following the meeting of the evaluation panel, the questions were pilot-tested to ensure that they were appropriate for Grade 12 (advanced level) students and free of flaws.

The Student Assessment Booklets were distributed to participating schools for administration in May 1990. The four different booklets were randomly distributed to students in each class so that approximately equal numbers of students within each class would be responding to each booklet. Although a time estimate for completion of the booklets was given, teachers were advised to allow students sufficient time to answer all questions. Students were excluded from the assessment only if they were enrolled in an English-as-a-second-language or a special-education program.

Questionnaires were answered by principals, teachers, and students during April 1990. Each student also answered one process problem at the same time that the Student Questionnaire was completed. Four different problems were randomly distributed to students in each class.

Students' answers to questions requiring a written response were marked by their classroom teachers. The scoring guide for these questions is given in *Mathematics, Grade 12 (Advanced Level): Review Instruments.*

3.3 Interpretation of Results

An interpretation panel consisting of teachers, administrators, university professors, trustees, and parents reviewed the test questions and results to establish standards and make judgements about student achievement. Panel members are listed in Appendix G.

Establishing Standards

The first task of the panel was to decide what constituted acceptable and desirable levels of performance on the test questions. This determination was based on the members' familiarity with skill levels of Grade 12 (advanced level) mathematics students and with the Ontario curriculum. When setting these standards, the panel members had no knowledge of students' actual performance levels in responding to the test questions.

To establish these standards, panel members reviewed each of the test questions prior to the tabulation of the results. Members were then asked to determine for each question the percentage of correct answers that would constitute (a) an acceptable level of performance and (b) a desirable level of performance. Acceptable performance was defined as the level of achievement below which a real weakness was indicated. Desirable performance was defined as a higher level of achievement that would indicate a strength. The desirable level was not to be considered an ideal level of achievement.

The panel carried out its task in three stages.

- **Stage 1:** Each panel member was instructed to set acceptable and desirable levels of performance for all questions in the assessment booklets and for the four process problems.

- **Stage 2:** Panel members met in small groups to compare their individual ratings, discuss their rationales for them, and arrive at a small-group consensus on the levels of performance for each question.

- **Stage 3:** Once the small groups had arrived at a group consensus, panel members met as one large group. The small-group ratings were shared, and a large-group consensus for acceptable and desirable levels of performance for all questions was established.

This process determined the standards by which assessment outcomes could be judged and rated.

Making Judgements

The mandate of the panel at the second phase of the interpretation process was to create overall ratings of student achievement by examining the standards established during the first phase and the scores achieved by students on each of the questions in the Student Assessment Booklets. For each question, overall student achievement was rated on the following scale:

5 – superior:	This level indicated outstanding student performance, showing a definite strength in the program.
4 – strong:	This level indicated strong student performance and is to be expected if the program has been implemented effectively.
3 – satisfactory:	This level indicated acceptable student performance and is to be expected if the program has been implemented adequately.
2 – marginal:	This level indicated student performance that is slightly less than satisfactory, and points out an area of the program that needs improvement but is not a major weakness.
1 – weak:	This level indicated student performance that is definitely unsatisfactory, and points out a major weakness in the program.

To establish ratings on the five-point scale, the panellists examined each question, the percentage correct on that question, and the acceptable and desirable levels set during the first phase. Once again, a multistage process was used to arrive at a group consensus.

- **Stage 1:** Each panellist assigned a performance rating to each question in the assessment booklets.

- **Stage 2:** The panellists met in small groups to share and discuss their individual ratings and to arrive at a small-group consensus on the rating for each question.

- **Stage 3:** Once the small groups had arrived at a consensus for the ratings on all questions, they met as a large group to arrive at a whole-group consensus for each question.

- **Stage 4:** Following the discussion and establishment of consensus on each question, the whole group was presented with the distribution of question ratings within each of the curriculum areas examined. On the basis of this distribution, the whole group arrived at a single descriptor of student performance for each curriculum area.

The panel generated considerable discussion and debate before a consensus emerged. This discussion assisted in identifying the strengths and weaknesses discussed in "8. Student Achievement".

3.4 Provincial Review Participants

Reviews of mathematics were conducted in both English and French schools. Two different groups were involved in the review: those chosen to constitute the provincial sample and those who opted to participate in board reviews. The present report pertains only to the English-language provincial sample. The results of the French review may be found in *Les mathématiques, 12e année (niveau avancé) : rapport à l'intention des éducateurs et des éducatrices.*

A sample of 99 schools was randomly selected from the entire population of English-language schools in Ontario offering mathematics at the advanced level in Grade 12. The data collected from this sample are representative of the entire Grade 12 population taking advanced-level mathematics. Students taking mathematics in the first semester, students in English-as-a-second-language programs, and students identified as exceptional by an Identification, Placement, and Review Committee did not participate. The number of respondents for each instrument is shown in Table 1. For Student Assessment Booklets, the number of respondents represents the total number of students for all booklets. Since some students answered more than one booklet, this number does not equal that for booklets reported in Appendix D.

Table 1
Number of Respondents

Instrument	*Respondents*
Principal Questionnaire	97
Teacher Questionnaire	175
Opportunity-to-Learn Form	171
Student Questionnaire	4851
Student Assessment Booklets	4529
Process Problems	4821

Response rates were 98 per cent for the Principal Questionnaire and 99 and 97 per cent for the Teacher Questionnaire and the Opportunity-to-Learn Form, respectively. Based on September enrolments, the response rates for student materials were 77 per cent for the Student Questionnaire and Process Problems, and 72 per cent for the Student Assessment Booklets.

3.5 Board Review Participants

Forty-eight school boards carried out their own local reviews, using the questionnaires and test booklets created for the provincial review. The board reviews involved all schools offering mathematics at the advanced level under the jurisdiction of the participating boards. School boards had the option of including exceptional students and students in ESL programs. The board results do not form part of the information provided in this report.

Every school participating in a board review or in the provincial sample was provided with its own results. Board results were provided to those boards that conducted board reviews.

4. Principal Questionnaire

A profile of the school environments in which Grade 12 mathematics is taught at the advanced level in Ontario can be derived from the information collected by the Principal Questionnaire. The location of the school, and the leadership, the quality of staff, and the use and deployment of resources within it, combine to define the educational framework within which the school operates. The complete Principal Questionnaire and the responses to each question are presented in Appendix A.

4.1 Enrolment

The average enrolment in the schools which formed the sample for the review was 991. Thirteen per cent of the schools had an enrolment of fewer than 600 students; 10 per cent had 1500 or more students. The average enrolment in Grade 12 mathematics at all levels was 164 students. The average enrolment in Grade 12 mathematics at the advanced level was 104 students. For schools in the sample, 65 per cent of the Grade 12 mathematics students were at the advanced level. For the province as a whole, this percentage would be considerably lower because schools offering only basic or general programs were not part of the sample.

4.2 The Location of School and Structure of Program

Seventy-one per cent of the schools served urban or suburban areas, 25 per cent served rural areas, and 4 per cent served an isolated area.

The average number of hours of mathematics instruction reported was 114 per year. Eighty-four per cent of principals reported an allocation of 110 to 119 hours to mathematics instruction and 15 per cent reported 120 or more hours.

Most of the mathematics courses in Grade 12 (advanced level) were semestered (77 per cent). Twenty-nine per cent of the schools offered Immersion or Extended French programs. Seven per cent of the schools offering these programs taught Grade 12 (advanced level) mathematics in French.

4.3 Frequency of Staff Meetings on Mathematics

The highest percentage of principals (40 per cent) reported that teachers met once every month or every second month to discuss mathematics programs. Twenty-three per cent reported that meetings were held once every two or three weeks, and 33 per cent reported that meetings were held once a week.

4.4 Use of Planning Documents

Approximately 90 per cent of principals reported that "a lot" of emphasis was placed on Ministry of Education guidelines and on school-level planning documents for planning the mathematics program. Thirty-one per cent said that no emphasis was placed on board documents.

4.5 Reviews of Mathematics Programs

Fifty-nine per cent of principals reported that their school boards had a procedure for regularly reviewing mathematics programs. A higher percentage (88 per cent) reported that their schools had a regular procedure. Seventy-one per cent said that a review had occurred in the previous school year (1988-89), and 16 per cent said there had been a review in the previous two to four years. These findings suggest that reviews are held on a frequent basis.

The primary emphasis in the reviews was on evaluation practices and planning documents, although teaching practices and resources received much attention. Student achievement measures were employed in 41 per cent of the mathematics reviews.

Principals reported that the major focus of change resulting from reviews was sequence and timing of programs (89 per cent reported a change). Changes in teaching strategies, teaching objectives, and student evaluation practices were also reported by approximately 70 per cent of principals.

4.6 Professional Development

Most principals reported that teachers engaged in professional development through professional reading, conferences, and seminars/workshops. Fifty-two per cent of principals said that teachers visited other classrooms, and approximately 40 per cent said that teachers took credit courses and used peer coaching/job shadowing for professional development.

5. Teacher Questionnaire

The Teacher Questionnaire and the responses to each question are presented in Appendix B. Teachers were asked background questions and questions about teaching methods, resources, and evaluation strategies.

5.1 Characteristics and Educational Background

A majority of teachers of Grade 12 (advanced level) mathematics teachers were male (69 per cent). Many had eleven or more years of teaching experience (80 per cent); 1 per cent had one year, 10 per cent had two to five years, and 9 per cent had six to ten years. Most of this experience was gained in secondary school: 93 per cent of teachers had not taught in elementary school. Seventy-seven per cent of teachers had eleven or more years of experience teaching secondary school, while 22 per cent had two to ten years of such experience.

Most teachers (92 per cent) had received some instruction in mathematics methods as part of their education, and 40 per cent had taken two or more full courses in mathematics methods. Eighty-six per cent of teachers reported taking five or more full-year university courses in mathematics.

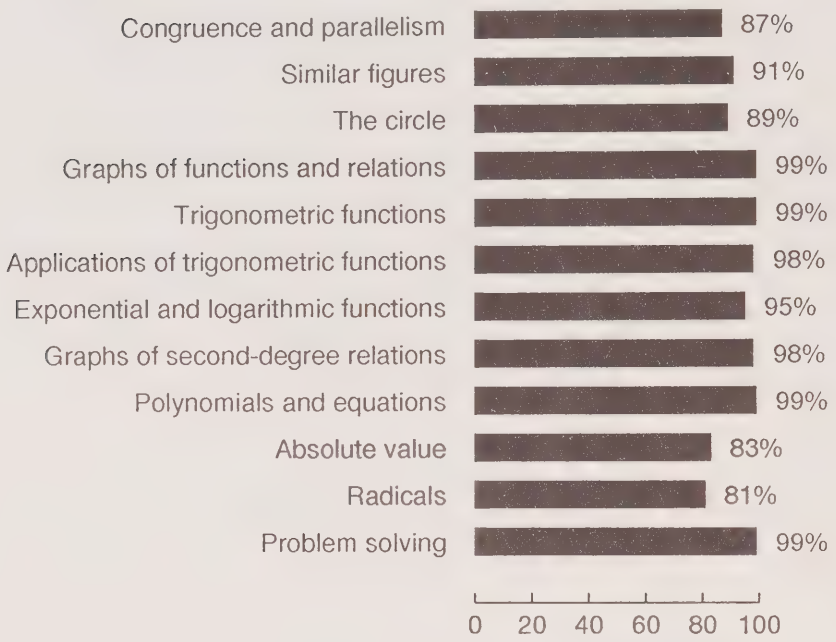
Forty-two per cent of teachers reported taking one course in measurement and evaluation or testing as part of their teacher training and/or graduate program. A high number (38 per cent) reported having taken no such course. More than half of the teachers (59 per cent) had not had training in the use of computers to teach mathematics.

5.2 Importance of Mathematics Topics

Teachers were asked to rate the importance of instruction of selected topics in mathematics. The percentage of teachers rating each topic as "important" or "very important" is presented in Figure 1. Most topics were rated as "important" or "very important" by more than 90 per cent of teachers. The topics receiving the lowest ratings were absolute value and radicals, which were rated as "important" or "very important" by 83 per cent and 81 per cent of teachers, respectively.

Figure 1
Relative Importance of Selected Mathematics Topics

Percentage of teachers rating each topic as "important" or "very important"



Percentages reported in the graph are based on raw data collected from the Teacher Questionnaire and may vary from the data summary reported for question 8 in Appendix B because of rounding.

5.3 Problem Solving

The average percentage of time devoted to the problem-solving process by teachers was 30 per cent. Teachers overwhelmingly (95 per cent) regarded problem solving as a component to be integrated into the mathematics program throughout the year, rather than as something to be taught as a separate unit. In teaching problem solving, they tried to provide students with a systematic approach (96 per cent) and to teach specific strategies (93 per cent). Eighty-nine per cent of teachers also encouraged students to develop their own strategies. Ninety-five per cent of teachers had students discuss their solutions with each other, and approximately 70 per cent had students write up their solutions or results and present them to the teacher or report their results to the whole class.

5.4 Homework

Thirty-nine per cent of teachers reported that their students would take two to three hours per week to complete mathematics homework. Twenty-nine per cent of teachers reported that the students would take three hours or more.

5.5 Use of Calculators and Computers

Approximately half (46 per cent) of the teachers reported that students had access to computers for mathematics. In the classrooms where computers were used, 57 per cent of teachers reported that less than 4 per cent of class time was given to having students use computers. When computers were available, teachers most frequently encouraged students to practise new concepts, to learn concepts and skills, and as an aid in doing projects. Computers were never used for tests.

Teachers reported that calculators were used often in mathematics classes, that is, during every or almost every class (75 per cent), or occasionally, that is, during about half of the lessons (20 per cent). Teachers encouraged students to use calculators during tests (94 per cent), to check answers to exercises (91 per cent), and as an aid in doing projects (82 per cent). This last category is interesting; on the Student Questionnaire, very few students reported doing projects in mathematics.

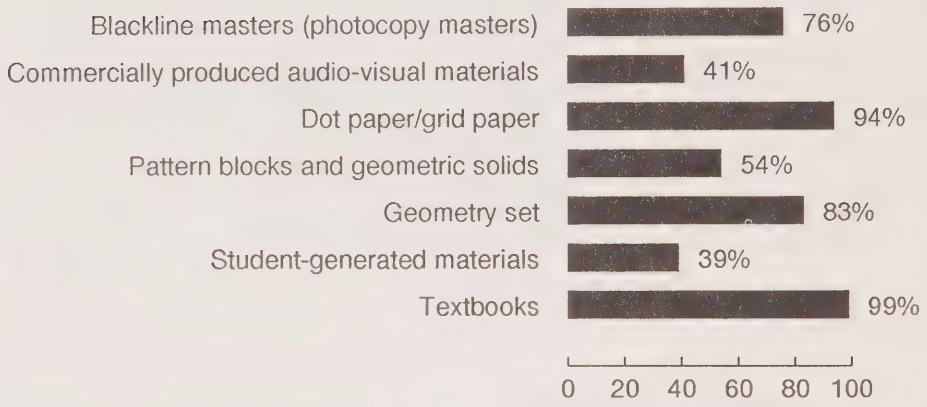
5.6 Instructional Materials

The two resources used most frequently by teachers in preparing lessons were textbooks (74 per cent reported "a lot") and materials prepared by themselves or other teachers in the school (66 per cent reported "a lot"). The Ministry of Education guideline received moderately high emphasis (58 per cent reported "a lot"). Approximately two-thirds of the teachers gave "some" emphasis to the use of suggestions from students, journals or other teacher reference books, and materials from in-service activities.

Textbooks, dot paper/grid paper, geometry sets, and blackline masters were the most readily accessible materials for teachers. Teachers emphasized textbooks most heavily in their instruction, followed by dot paper/grid paper, geometry sets, and blackline masters. Approximately 60 per cent of teachers said that they placed no emphasis on student-generated materials or on commercially produced audio-visual materials. The percentage of teachers who reported placing "some" or "a lot" of emphasis on each resource is presented in Figure 2.

Figure 2
Emphasis Placed on Resources for Teaching Mathematics

Percentage of teachers who placed "some" or "a lot" of emphasis on each resource



Percentages reported in the graph are based on raw data collected from the Teacher Questionnaire and may vary from the data summary reported for question 18 in Appendix B because of rounding.

5.7 Teaching Methods

On average, teachers reported that they used a chalkboard and/or projector 57 per cent of the time. On average, teachers had students working individually for 25 per cent of the time. Teacher demonstrations with concrete materials were used to a lesser extent (an average of 8 per cent of the time.)

Thirty-three per cent of teachers reported that students worked in groups 10 to 19 per cent of the time, and 21 per cent reported that students worked in groups 20 to 29 per cent of the time.

5.8 Evaluation

Teachers were asked what percentage of students' marks was determined through the use of various evaluation tools, including quizzes or short tests, cumulative or end-of-unit tests, formal examinations, observation of student behaviour, peer/self-evaluation, and projects and assignments. Cumulative or end-of-unit tests seemed to bear the most weight, along with formal examinations. Together, they constituted approximately 80 per cent of the final mark. Observation of student behaviour does play a part in evaluation, as does peer/self-evaluation, although these tools carry less weight. On average, quizzes determined 11 per cent of a student's mark, and projects and assignments 6 per cent.

6. Student Questionnaire

The Student Questionnaire and the student responses to each question are presented in Appendix C. Students were asked a series of questions to determine their attitudes towards or enjoyment of various types of mathematics questions, followed by questions on classroom procedures; on the significance of mathematics to their lives, both in and out of school; and on ways in which they deal with mathematics problems that they do not initially understand.

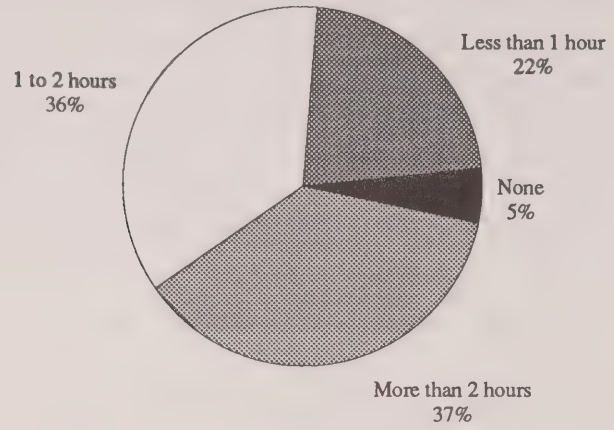
6.1 Background Characteristics

The sample surveyed included a somewhat larger number of males (52 per cent) than females (48 per cent). Sixty-five per cent of all mathematics students in the Grade 12 sample schools were studying mathematics at the advanced level, as reported in the Principal Questionnaire. The actual percentage for the province would be lower because schools that do not offer mathematics at the advanced level were not part of the sample. Sixty-four per cent of students sampled were born in 1972, 20 per cent in 1973, and 14 per cent in 1971 or before. A high percentage (86 per cent) of students declared English to be the language of the home.

6.2 Mathematics Activities Outside School

As shown in Figure 3, most of the Grade 12 students in mathematics at the advanced level reported doing either more than two hours of mathematics homework each week (37 per cent) or one to two hours per week (36 per cent). Twenty-two per cent did less than one hour per week.

Figure 3
Amount of Mathematics Homework Done Each Week



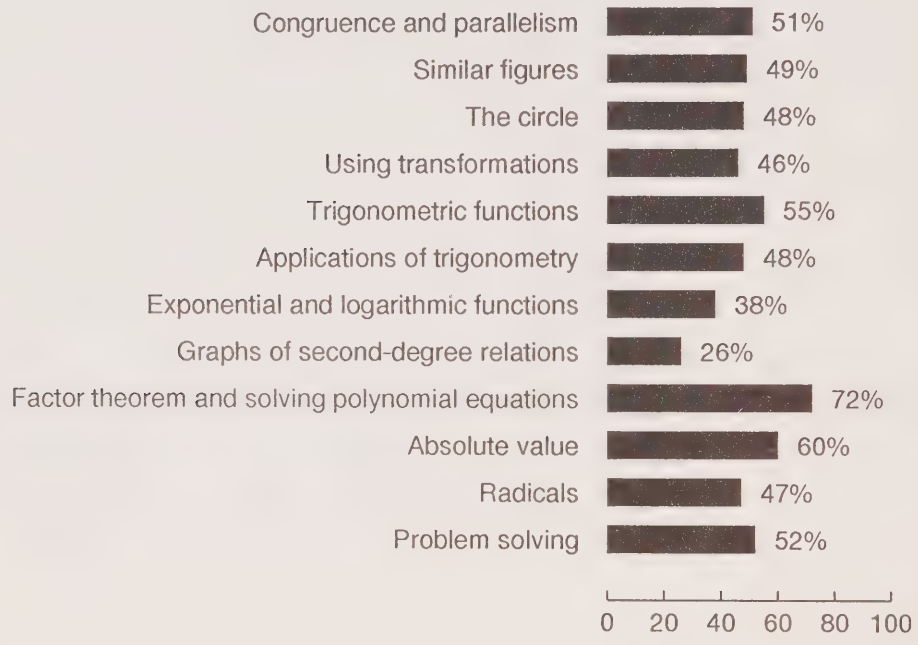
Students reported spending nearly as much time watching television in one day as they did doing mathematics homework in one week. Thirty-seven per cent reported watching for one to two hours per day, and 28 per cent reported watching for more than two hours per day.

When mathematics is done at home, 24 per cent of students reported receiving occasional help from a family member. Seventy-two per cent of students reported that never, or almost never, did they receive help from a family member. This finding relates to a question, posed later in the questionnaire, in which students were asked whether they would wait and ask someone at home if they did not understand a mathematics problem. Seventy-three per cent said they would never wait, and 23 per cent said they would sometimes wait. This finding suggests that students do not regard or use family members as a main resource for mathematics problem solving or reinforcement. Mathematics activities related to the school program appear to be tied to the school day.

6.3 Student Attitudes Towards Topics in Mathematics

Students were asked whether they liked doing questions dealing with twelve specific mathematics topics. The percentage of students who said they liked doing questions in a specific topic is given in Figure 4. The majority of students liked doing questions that deal with congruence and parallelism, trigonometric functions, factor theorem and solving polynomial equations, absolute value, and problem solving. The topics with the highest response of "We do not do these kinds of questions" were exponential and logarithmic functions and graphs of second-degree relations.

Figure 4
Percentage of Students Who Like Each of Twelve Topics in Mathematics



6.4 What Happens in Mathematics Classes

Students were asked about teaching procedures and student activities in the classroom. Student responses indicate that class time seems to be devoted primarily to presentations to the whole class as a group and work performed by the individual student. Seventy per cent said that they did mathematics on their own in class almost every day. Most students (77 per cent) said that they used a scientific calculator almost every day. Eighty-six per cent of students said that they never used a computer. Details of responses to questions about classroom activities are provided in Table 2.

Table 2
Percentage of Students Reporting Frequencies of Classroom Activities in Mathematics

	Percentage of Students			
	Almost every day	*Once a week or so*	*Only once in a while*	*Never*
How often do you do mathematics at the board or on the overhead projector?	27	11	31	31
How often is mathematics presented to the whole class as a group?	91	2	3	3
How often do you do mathematics in small groups?	9	8	31	52
How often do you do mathematics on your own in class?	70	12	15	4
How often do you do projects on mathematics topics?	1	3	30	67
How often do you do projects in other subjects that involve the use of mathematics?	11	14	53	22
How often do you write tests or quizzes in mathematics?	3	67	29	0
How often do you use a scientific calculator to do mathematics in class?	77	13	9	2
How often do you use a computer to do mathematics in class?	1	1	11	86

More than a third of the students said that they worked at the board or on the overhead projector at least once a week or almost every day.

Doing mathematics in small groups did not seem to be a preferred approach. Fifty-two per cent of students responded that they never worked in this way; 31 per cent responded that they worked in this way once in a while. Projects in mathematics were also rare; 67 per cent of

students reported never doing projects and 30 per cent reported doing projects only once in a while. There was little overlap between Grade 12 (advanced level) mathematics and other subjects, as 53 per cent of students reported working on projects involving mathematics in other subjects only once in a while and 22 per cent said they never did such projects. However, 25 per cent reported working on such projects almost every day or once a week.

A majority of students (67 per cent) reported writing tests or quizzes once a week or so; 29 per cent reported doing them only once in a while.

6.5 Use of Calculators and Computers

Most students (77 per cent) reported using a scientific calculator to do mathematics in class almost every day, and an even higher percentage (81 per cent) reported using calculators at home. Whereas calculators are well established as an aid in Grade 12 (advanced level) mathematics, computers are not. Most students (86 per cent) reported that they never used a computer to do mathematics in class. Fifty-six per cent of students reported that they had a computer at home, but only 4 per cent reported using the computer for mathematics school work.

6.6 How Students Feel About Mathematics

Students' perceptions of the significance of mathematics both inside and outside the classroom and students' attitudes towards mathematics were the focus of a series of questions. Most students (93 per cent) agreed that mathematics was for everyone equally, although a small group (5 per cent) thought it was more for males than females and another small group (1 per cent) believed the reverse. Most students (73 per cent) thought that mathematics helped them outside of school. Fewer than half of the students (37 per cent) felt that learning mathematics involves mostly memorizing, and a high number (78 per cent) felt that knowledge of mathematics was important in order to get a good job.

Seventy-one per cent of students agreed or strongly agreed that they liked mathematics. Approximately half of the students thought that they were good at mathematics, and 33 per cent thought that mathematics was difficult. Student responses to the attitude statements are presented in Table 3.

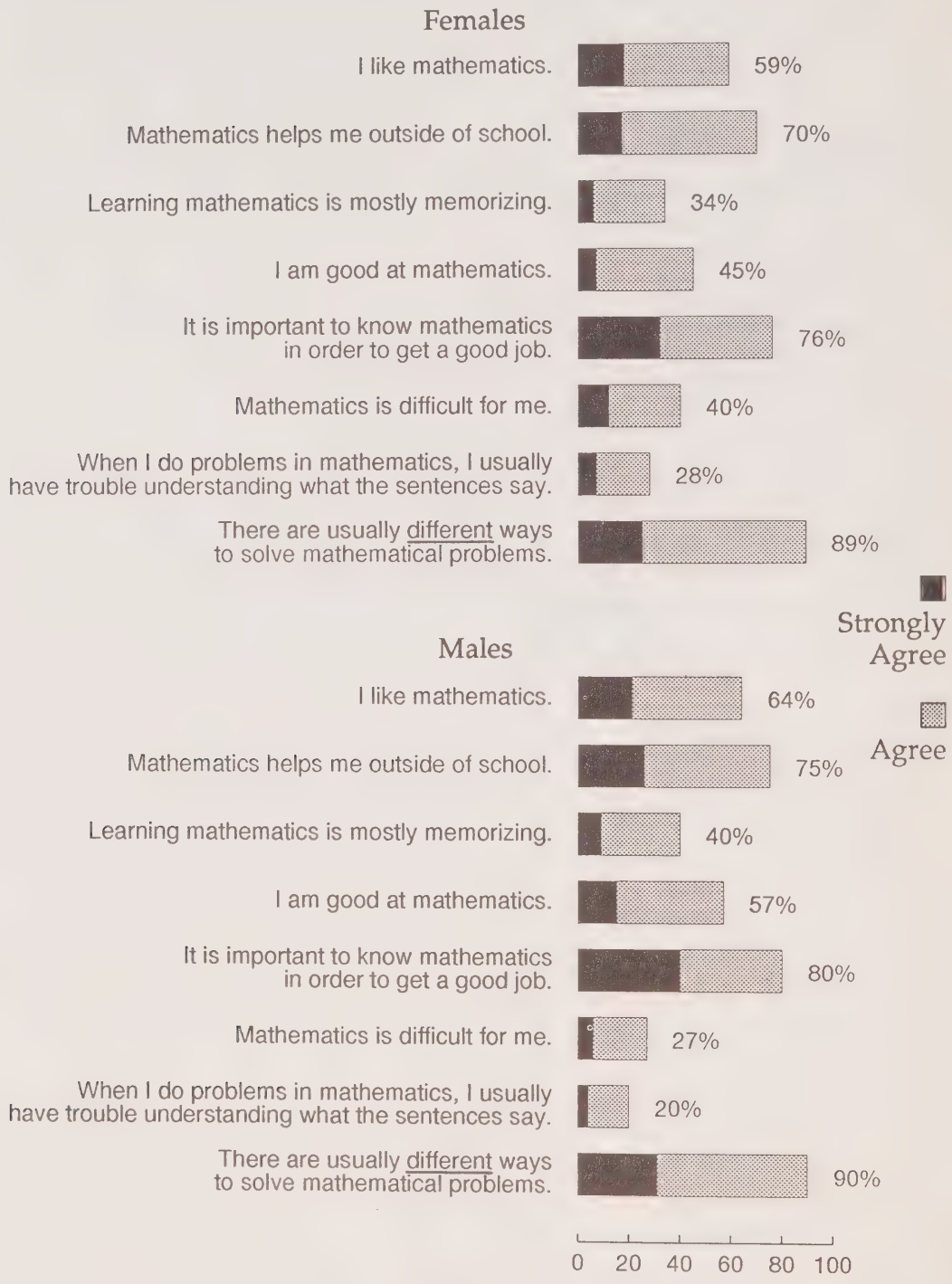
Table 3
Student Responses to Specific Attitudes Towards Mathematics

	Percentage of Students				
	SA	*A*	*NS*	*D*	*SD**
I like mathematics.	19	42	18	14	6
Mathematics helps me outside of school.	22	51	14	11	2
Learning mathematics is mostly memorizing.	7	30	13	41	9
I am good at mathematics.	11	40	25	17	6
It is important to know mathematics in order to get a good job.	36	42	11	9	2
Mathematics is difficult for me.	9	24	19	36	12
When I do problems in mathematics, I usually have trouble understanding what the sentences say.	5	18	18	46	13
There are usually different ways to solve mathematical problems.	28	62	8	2	0

**SA* = Strongly Agree; *A* = Agree; *NS* = Not Sure; *D* = Disagree; *SD* = Strongly Disagree

Student attitudes towards mathematics differed in some respects, according to gender. Results by gender are shown in Figure 5. A larger percentage of male students said that they were good at mathematics. A higher percentage of female students reported that mathematics was difficult for them. A slightly larger percentage of male students than female students said that they liked mathematics.

Figure 5
Gender Differences in Student Attitudes Towards Mathematics



The percentage of students selecting strongly agree and agree in response to these opinion statements, by gender, is given Table 4.

Table 4
Student Responses to Attitude Statements by Gender

	Percentage of Students			
	Female		*Male*	
	SA	*A*	*SA*	*A**
I like mathematics.	18	41	21	43
Mathematics helps me outside of school.	17	53	26	49
Learning mathematics is mostly memorizing.	6	28	9	31
I am good at mathematics.	7	38	15	42
It is important to know mathematics in order to get a good job.	32	44	40	40
Mathematics is difficult for me.	12	28	6	21
When I do problems in mathematics, I usually have trouble understanding what the sentences say.	7	21	4	16
There are usually different ways to solve mathematical problems.	25	64	31	59

**SA* = Strongly Agree; *A* = Agree

6.7 Solving Mathematics Problems

There were two statements on problem solving among the attitude statements reported in Tables 3 and 4 and Figure 5. Fewer than half of the students said that they had trouble understanding the sentences used to present problems. Most students (90 per cent) recognized that there are usually different ways to solve mathematical problems.

Table 5 tabulates the percentage of responses of students to a series of questions on how they dealt with problems that they did not understand. Most students (94 per cent) reported that they sometimes or often sought help from other students or tried to solve such problems on their own. Eighty-nine per cent said that they asked the teacher for help.

A fairly high percentage of students (65 per cent) reported that they sometimes gave up on problems, although 25 per cent said that they never did. Few students sought help at home. This finding is to be expected as most parents would not be able to do mathematics at this advanced level.

Figure 5
Specific Methods of Dealing With Difficult Mathematics Problems

When you don't understand a problem in mathematics, do you:	*Percentage of Students*		
	Often	*Sometimes*	*Never*
Try different ways to solve it on your own?	40	54	5
Ask another student for help?	38	56	6
Ask your teacher for help?	22	67	11
Wait and ask someone at home?	4	23	73
Just give up?	9	65	25

7. Planning Documents

As part of the assessment of the intended curriculum for the Grade 12 (advanced level) mathematics program, courses of study were collected from all schools and boards involved in the provincial sample. These documents were then analysed by a document analysis team of teachers. The purpose of this analysis was to describe what planning documents teachers of Grade 12 (advanced level) mathematics refer to in preparing their daily lesson plans.

The courses of study that were analysed varied widely in both depth and detail. Some were developed for system-wide use in all schools within a board; others were developed by teachers and/or department heads at a school for use in their own classes. Generally, it was found that, if the course of study had been developed at the board or system level, it was more likely to contain a statement of philosophy, a rationale, and a set of goals; if it had been developed by a teacher for only that teacher's use, it was more likely to be a functional document consisting of a list of required teaching topics, with an emphasis on knowledge and skills.

In some instances, partial documents were sent in for analysis. For example, a curriculum document on senior mathematics that included courses of study for all levels and grades in the senior division would have been disassembled and the Grade 12 (advanced level) section submitted separately for analysis. In such a case, the document analysis team was unable to determine whether sections of the document that had been omitted, or its introduction, might have included general information on the teaching of mathematics, evaluation of the program, goals of the senior mathematics program, and so on. Furthermore, special documents on evaluation produced by some boards were not submitted because they were not part of the Grade 12 (advanced level) mathematics course of study. Because of these anomalies, the data gathered from the planning documents should be interpreted broadly. Appendix F contains a detailed summary of the data compiled by the document analysis team.

7.1 Type of Document

Documents were of two types: board level (produced by school boards and designed to be used in all Grade 12 [advanced level] mathematics classes), and school level (produced in a school for use in only that school). Of the Grade 12 courses of study analysed, nineteen were board level and ninety-six were school level.

7.2 Rationale/Objectives

Half of the school-level courses of study contained a statement of philosophy or rationale. Seventy-nine per cent of the board-level documents included such a statement. Specific objectives of the Grade 12 (advanced level) mathematics program were included more frequently: in 73 per cent of school-level documents and 84 per cent of board-level documents.

The analysis team looked for three types of objectives: skills, knowledge, and attitudes. Skills and knowledge objectives were included in almost all Grade 12 courses of study; attitudinal objectives in fewer cases. For example, 90 per cent of board-level documents listed objectives for skills and 100 per cent for knowledge, while only 21 per cent listed attitudinal objectives.

7.3 Content Areas

All the documents included in this analysis were based upon the curriculum guideline *Mathematics: Intermediate and Senior Divisions, 1985. Part Three: Grades 11 and 12, Advanced Level.* The guideline lists both compulsory and optional topics. For the most part, board and school documents included all the compulsory topics. Approximately half of the documents listed the optional topics.

Table 6 lists the compulsory and optional topics from the guideline. It shows, for each category, the percentage of documents that included the topic listed.

Table 6
Percentage of School- and Board-Level Documents Dealing With Guideline Topics

Guideline topics	*School-level documents*	*Board-level documents*
Geometry		
Congruence and parallelism	96	100
Similar figures	96	100
The circle	95	100
Relations and Functions		
Graphing functions and relations	97	100
Trigonometric functions	99	100
Applications of trig functions	99	100
Exponential and log functions	99	100
Graphs of second-degree relations	99	100
Algebraic Operations		
Polynomials and equations	99	100
Absolute value	96	95
Radicals	91	90
Statistics		
Dispersion (optional)	23	53
Correlation (optional)	23	47

7.4 Teaching Methods

The majority of board-level documents included some suggestions on teaching methods: they provided specific instructional techniques, learning strategies, lesson plans, sample tasks for students, and various teaching approaches to Grade 12 (advanced level) mathematics. Instructional methods or teaching strategies were included in 68 per cent of board-level documents and 40 per cent of school-level documents.

7.5 Calculators and Computers

By the time students have reached Grade 12, use of the calculator for mathematics is virtually universal. Mentioning it in a course of study might therefore seem unnecessary. Use of the calculator, however, was specifically indicated in 32 per cent of board-level documents and 25 per cent of school-level documents. While students have access to computers in most schools in Ontario, student use of them is quite low, according to the Student Questionnaire. Courses of study discuss computer use in 21 per cent of board-level documents and 14 per cent of school-level documents

7.6 Evaluation Practices

Many boards in Ontario have produced documents dealing specifically with evaluation. Even so, many of the planning documents included additional information for teachers about evaluation related to the Grade 12 (advanced level) mathematics program. Where evaluation was included in the document, the traditional modes of evaluation, such as unit tests, post-tests, examinations, essays, and projects, were most often stressed. Observation of student attitude and student performance was, nevertheless, included in 60 per cent of the school-level courses of study.

7.7 Conclusion

An analysis of the planning documents for Grade 12 (advanced level) at the school and board levels reveals that boards have historically invested enormous amounts of energy and resources into curriculum development. It also shows that Grade 12 (advanced level) mathematics teachers do move towards newer teaching and evaluation methods, but do so cautiously, and without giving up the more traditional methods.

8. Student Achievement

A number of instruments were used to determine whether or not students were achieving the goals of the Grade 12 (advanced level) mathematics program. They included multiple-choice questions, questions requiring short answers, and problems requiring extended responses. Students were allowed to use calculators while working on the questions. The following areas of the curriculum were covered by the instruments: geometry, relations and functions, algebraic operations, and process problems. The questions measured students' ability to recall, understand, and apply mathematical facts, skills, and concepts; to analyse problems; and to select and carry out a strategy to solve an unfamiliar problem. More detailed descriptions of the assessment instruments and curriculum areas are presented in "3. Description of the Measures and Samples for the Provincial Review".

The results were reviewed by an interpretation panel, which rated student performance on the following five-point scale:

5 – superior
4 – strong
3 – satisfactory
2 – marginal
1 – weak

Student achievement for half the curriculum areas in Grade 12 (advanced level) mathematics was rated as *marginal* or *weak*. Student performance was rated as *strong* only in the topic of similar figures. Polynomials and equations, congruence and parallelism, the circle, and radicals were topics for which student performance was rated as *satisfactory*.

Some areas of concern, where improved performance would be desirable, were graphing functions and relations, trigonometric functions, applications of trigonometric functions, and exponential and logarithmic functions. The performance on the topic of graphs of second-degree relations was judged to be *weak*.

Questions for each of the areas of the curriculum are presented in Appendix D, along with the percentage of students answering each question correctly and the ratings of the interpretation panel (Tables D-1 to D-12). The results for each curriculum area are summarized in Table D-15.

The four Student Assessment Booklets were randomly distributed within each class so that one-quarter of the students in the provincial sample answered each booklet. Since each student did not answer all questions, there is a degree of uncertainty associated with the provincial statistics (sampling error). The magnitude of this uncertainty for each curriculum area and for individual questions is specified in Appendix E in the form of confidence-band widths. For example, the confidence-band width associated with the mean percentage correct for geometry was ± 0.6, and the reported mean percentage correct was 52.2. The confidence band for geometry was, therefore, 51.6 to 52.8 per cent. The mean percentage correct would fall within this range 95 per cent of the time if the review were repeated with a similar sample.

In the presentation of results, achievement is reported as the average percentage correct for the questions on each topic. Ratings provided by the interpretation panel are also presented. Since the questions on one topic might be considerably more difficult than the questions on another topic, it is not advisable to compare the results on the different topics solely on the basis of average percentage correct. When the interpretation-panel members reviewed the results, they considered the difficulty of the question, their expectations of what Grade 12 (advanced level) mathematics students should be able to do, and the students' actual achievement. On that basis, the panel rated strength and weakness.

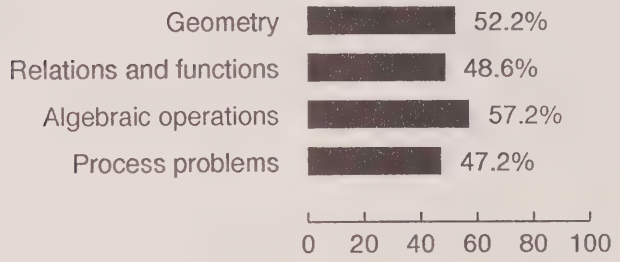
Table 7 indicates the percentage of questions within each area of the curriculum for which student performance was given a particular rating by the interpretation panel. For example, performance on 19 per cent of questions (five of twenty-seven) on geometry was rated as *strong*. The panel rated performance as *satisfactory* on 25 per cent of the relations and functions questions, and as *marginal* on 50 per cent of the algebra questions. Performance on one of four process problems was rated as *superior*.

Table 7
Percentage of Questions Within Each Curriculum Area Receiving Each Panel Rating

	Percentage of Questions			
Rating	*Geometry*	*Relations and functions*	*Algebraic operations*	*Process problems*
Superior	3	0	0	25
Strong	19	6	6	0
Satisfactory	41	25	33	25
Marginal	22	47	50	25
Weak	15	23	11	25

A summary of the average percentage correct for each curriculum area is presented in Figure 6.

Figure 6
Summary of Mathematics Achievement (Percentage Correct) by Curriculum Area



The panel ratings were: *satisfactory* for geometry, *marginal* for relations and functions, *satisfactory* for algebraic operations, and *satisfactory* for process problems.

The interpretation panel was of the opinion that the guideline for Grade 12 (advanced level) mathematics needs to be revised to reduce the amount of content and to reflect modern technology. There is a need for more coherence in the course, and greater emphasis on the development of higher thinking skills. The members of the panel also suggested that the results of this review be considered in the context of the course's being in the early stages of implementation. The panel recommended that future reviews give more emphasis to process components.

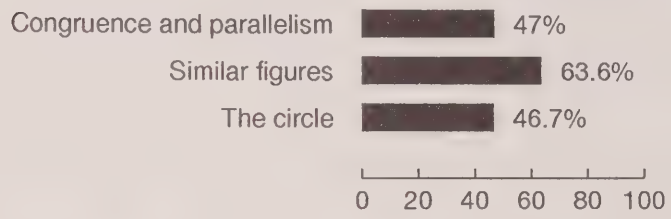
The average percentage correct for male and female students is given in Figure 7. Achievement levels are slightly higher for male students than for female students for all topics. The enrolment in Grade 12 mathematics at the advanced level was 48 per cent female and 52 per cent male.

Figure 7
Average Student Achievement for Each Curriculum Area by Gender

8.1 Geometry

By the end of the Grade 12 (advanced level) mathematics, students are expected to solve numerical problems and prove simple deductions involving congruent triangles, parallel lines, similar figures, and circles. They should also be familiar with the features of indirect proof. The results for achievement in geometry are shown in Figures 8 and 9.

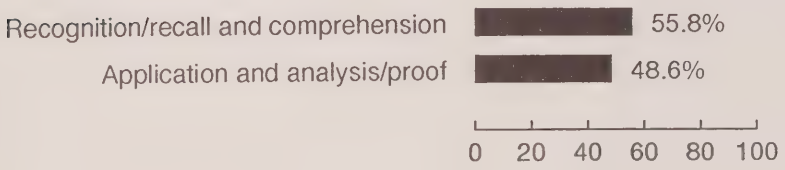
Figure 8
Student Achievement in Geometry by Topic



The interpretation-panel ratings were: *strong* for similar figures, and *satisfactory* for the other two topics.

Questions were classified according to the following cognitive levels (described in "3.1 The Instruments"): recognition or recall and comprehension, application, and analysis or proof. As there were only a few questions testing analysis or proof, achievement on these questions was combined with achievement on questions testing application.

Figure 9
Student Achievement in Geometry by Cognitive Level



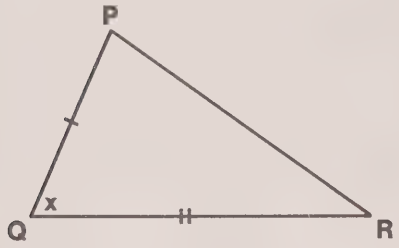
Congruence and Parallelism

There were ten questions on this topic, and the interpretation panel rated the results as *satisfactory*. The results on three of the questions were rated as *strong*, and on three as *satisfactory*. The results on two questions were rated as *marginal*, and on two as *weak*. The interpretation panel felt that question 9 did not test a guideline objective.

Question 4 (reproduced below) was one of the questions for which performance was rated as *strong*. It tests students' understanding of the properties of congruence.

Question 4

Examine the triangle below:



Student Responses

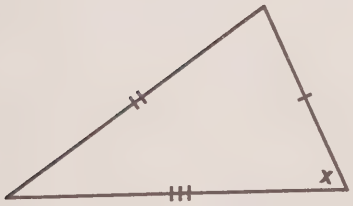
*86%	A)
1%	B)
7%	C)
5%	D)

A triangle which is congruent to ΔPQR is:

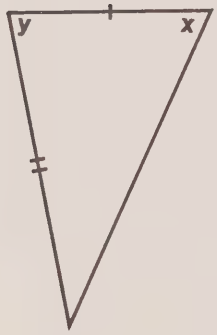
A)

B)

C)



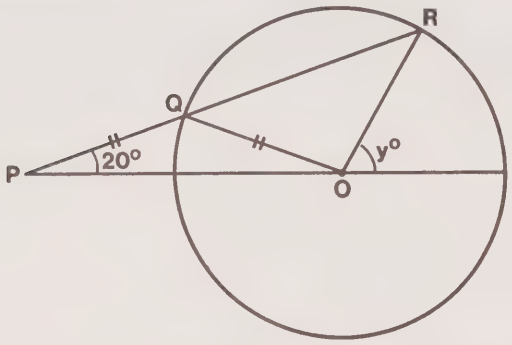
D)



Students demonstrated an ability to use geometric properties to solve numerical problems. The performance on question 129 (reproduced below) was 67 per cent; this result was rated as *satisfactory*.

Question 129

PQR is a secant to the circle below with centre O and PQ = OQ.



The value of y is:

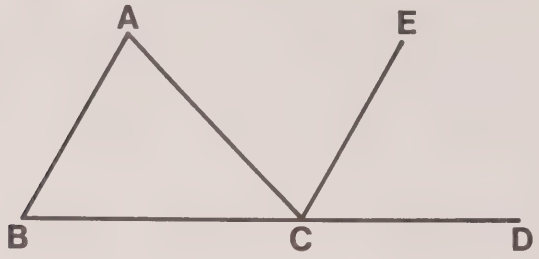
Student Responses

14%	A)	40
*67%	B)	60
12%	C)	80
5%	D)	140
3%		omit

Questions 25 and 145 tested students' ability to develop a geometric proof. The performance on question 25 (reproduced below) was rated as *marginal* and that on question 145 as *weak*.

Question 25

Given ΔABC with BC produced to D. EC bisects $\angle ACD$ and EC $\parallel$ AB.



Prove: ΔABC is isosceles.

Student Responses

31%	Selects an appropriate strategy and produces a proof
5%	Selects an appropriate strategy, but makes a minor error
16%	Makes a major error or does not complete
24%	Shows little understanding
24%	omit

The interpretation panel suggested that emphasis should continue to be given to having students develop convincing arguments in the form of proofs.

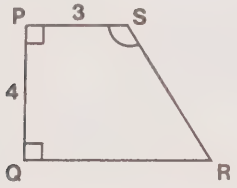
Similar Figures

There were nine questions on this topic and the overall rating by the interpretation panel was *strong*. The results were rated as *superior* for one question, *strong* for three questions, *satisfactory* for three questions, *marginal* for one question, and *weak* for one question.

Question 87 (reproduced below) was the question for which the performance was rated as *superior*. Ninety-two per cent of students gave the correct response.

Question 87

Examine quadrilateral PQRS:

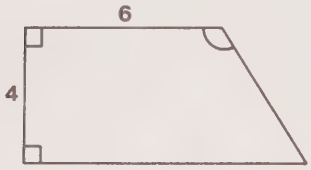


Student Responses

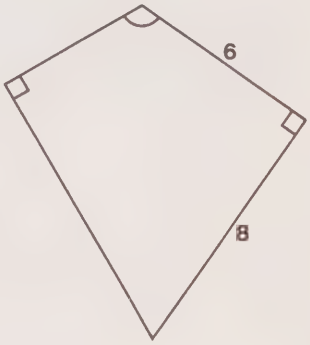
4%	A)
1%	B)
*92%	C)
3%	D)

Which quadrilateral is similar to PQRS?

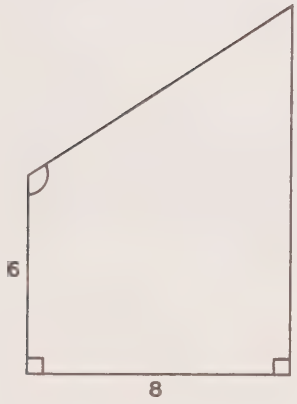
A)



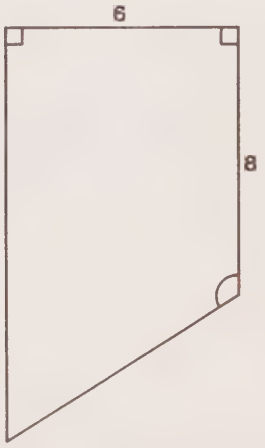
B)



C)



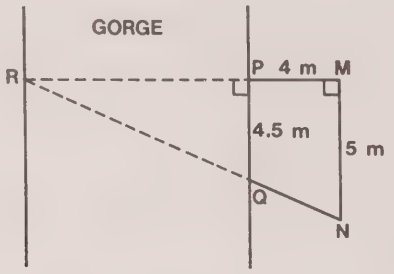
D)



The student performance of 62 per cent correct for question 6 (reproduced below) was rated as *satisfactory*.

Question 6

During a course in orienteering, you have to determine the distance across a gorge. R is a rock on one edge of the gorge. You position markers P, Q, M, and N as shown in the diagram.



If P and Q are on the edge of the gorge, then the distance PR across the gorge is:

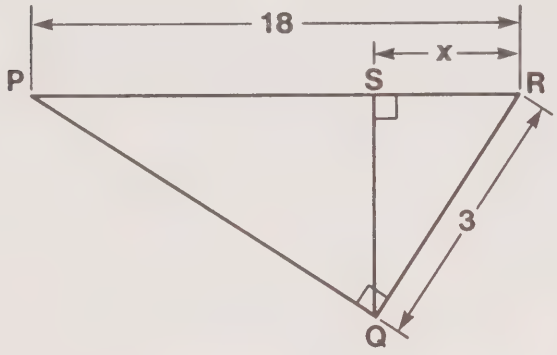
Student Responses

3%	A)	3.2 cm
21%	B)	3.6 cm
62%	C)	36.0 cm
*9%	D)	40.0 cm
5%	omit	

The performance on question 7 (reproduced below) was rated as *weak*. The difficulty that students had with this question is probably an indication of a weakness in algebra rather than in geometry.

Question 7

In the diagram below, PR = 18, QR = 3, and SR = x.



x is equal to:

Student Responses

32%	A)	2
19%	B)	$\frac{3}{2}$
9%	C)	1
*36%	D)	$\frac{1}{2}$
4%	omit	

The Circle

The overall rating for this topic was *satisfactory*. The results were rated as *satisfactory* on five of the nine questions, *marginal* on three, and *weak* on one.

Question 131 (reproduced below), which tested the ability to convert angles from radians to degrees, was one of the questions for which the rating was *satisfactory*.

Question 131

The measure of an angle in radians is $\frac{11}{3}\pi$. The measure of the angle in degrees is:

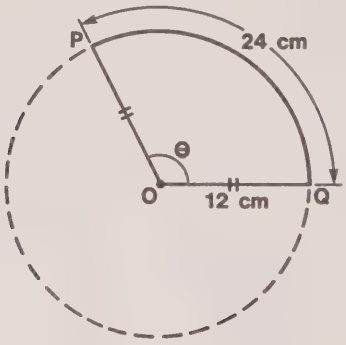
Student Responses

5%	A)	1 320
3%	B)	733
*68%	C)	660
22%	D)	11.5
2%	omit	

While students could carry out this conversion, the results for question 8 (reproduced below) – 30 per cent correct – led to a *weak* rating. The interpretation panel suggested that more emphasis should be placed on the concept of radian measure than on the process of conversion.

Question 8

Examine the diagram below:



The measure of $\angle\theta$ in radians is:

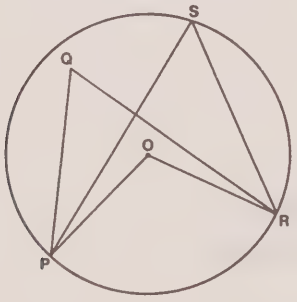
Student Responses

10%	A)	$\frac{1}{2}$
*30%	B)	2
39%	C)	$\frac{2}{\pi}$
17%	D)	2π
5%	omit	

The results on question 46 (reproduced below), which were rated as *marginal*, indicated a lack of understanding of a fundamental definition of the geometry connected with the circle.

Question 46

The circle below has centre O.



The inscribed angle(s) on arc PR shown in the diagram is (are):

Student Responses

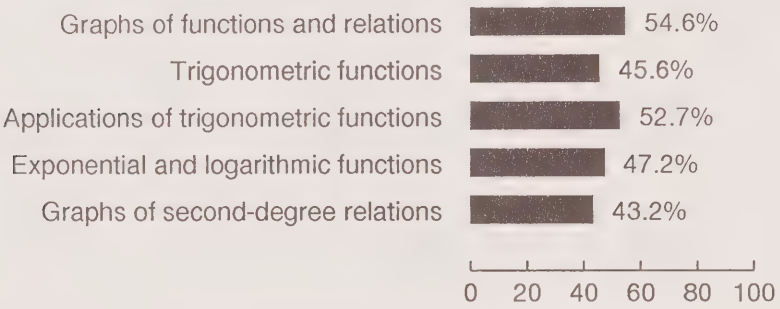
*31%	A)	$\angle$PSR only
5%	B)	$\angle$PQR only
24%	B)	$\angle$PSR and $\angle$PQR
39%	D)	$\angle$PSR, $\angle$PQR, and $\angle$POR
1%	omit	

8.2 Relations and Functions

This curriculum area includes graphing functions using transformations; the concept and graph of the inverse of a function; the graphs of trigonometric functions; the amplitude, periodicity, and phase shift of sinusoids; applications of trigonometric functions to problems involving triangles; the proofs of simple identities; and the solution of trigonometric equations. It also includes the exponential and logarithmic functions and graphs of some second-degree relations.

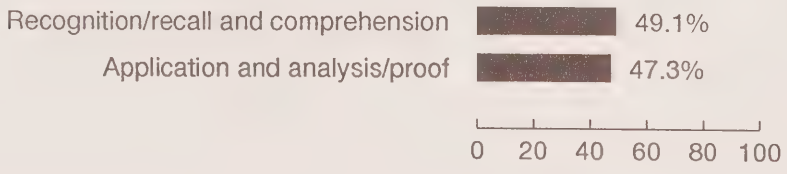
The average percentage correct for questions by topic is shown in Figure 10.

Figure 10
Student Achievement in Relations and Functions by Topic



The interpretation panel rating for student achievement on relations and functions was *marginal*. Four of the topics received a *marginal* rating, and the topic graphs of second-degree relations had a *weak* rating. The achievement of students by cognitive levels is given in Figure 11.

Figure 11
Student Achievement in Relations and Functions by Cognitive Level



Graphing Relations and Functions

Of the eight questions on this topic, the results were rated as *strong* on one, *satisfactory* on two, *marginal* on three, and *weak* on two. The overall rating was *marginal.*

Student achievement on question 10 (reproduced below) was rated as *strong*. This question tested students' knowledge of function notation.

Question 10

If $f(x) = 3 - x^2$ and $g(x) = \sqrt{6-x}$, then the value of $g((1))$ is:

Student Responses

*76%	A)	2
6%	B)	−2
11%	B)	$\sqrt{5}$
4%	D)	5
2%	omit	

When function notation was used with a variable rather than a specific value, the achievement fell. Only 45 per cent of students answered question 90 (reproduced below) correctly, a performance level that received a *marginal* rating from the interpretation panel.

Question 90

The functions **f** and **g** are defined by $f(x) = x - 1$ and $g(x) = (x + 3)^2$. Therefore $g(f(x))$ is equal to:

Student Responses

30%	A)	$(x - 1)(x + 3)^2$
21%	B)	$(x + 3)^2 - 1$
3%	C)	$(2x - 2)^2$
*46%	D)	$(x + 2)^2$
1%	omit	

Students' ability to recognize the equation of a function given the transformations applied to the graph in a standard position was tested by question 48 (reproduced below). The results were rated as *satisfactory*.

Question 48

The graph of $y = f(x)$ is vertically stretched by factor 2, then horizontally shifted left by 3 units. Which of the following is the equation of the resulting graph?

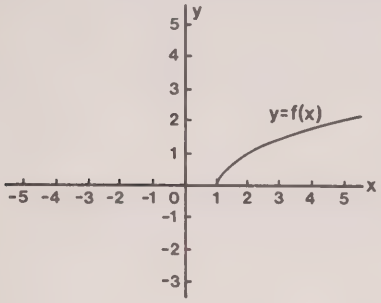
Student Responses

15%	A)	$y = 2f(x - 3)$
8%	B)	$y = f(2x) - 3$
13%	C)	$y = f(\frac{1}{2}x + 3)$
*64%	D)	$y = 2f(x + 3)$

One of the questions for which performance was rated as *weak* was question 141 (reproduced below) which tested students' knowledge of the reflection of a graph in the y-axis. The interpretation panel recommended that more emphasis should be placed on identifying the relationships between the transformations of the graph of a function and the corresponding equations. Achievement was rated as *marginal* on question 98, which was also on transformations.

Question 141

Examine the diagram below:

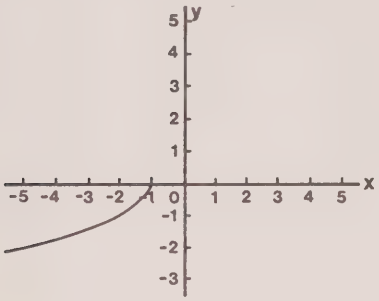


Student Responses

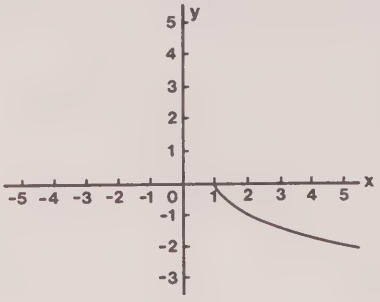
14%	A)
36%	B)
*36%	C)
12%	D)
2%	omit

Which of the following is the graph of y = f(– x)?

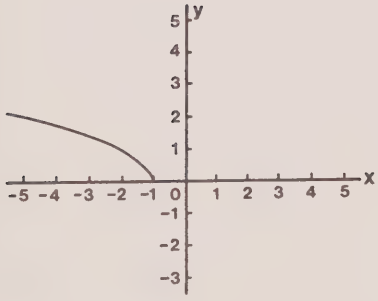
A)



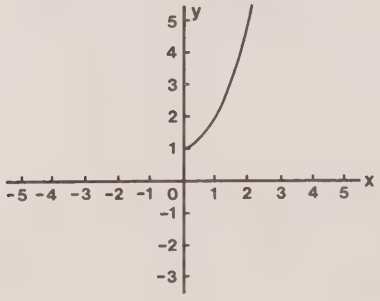
B)



C)



D)



Trigonometric Functions

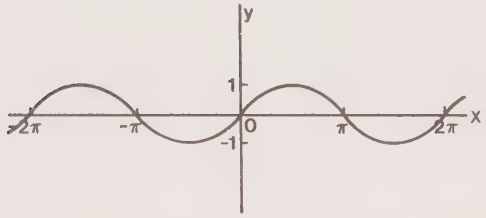
This topic includes the definition of the three basic trigonometric functions and their graphs, together with similar concepts for the reciprocal functions. The graphs of other sinusoids are related to the graphs of the sine and cosine functions through the consideration of transformations. The concepts of periodicity, amplitude, and phase shift are introduced.

Student performance received an overall rating of *marginal*. The results on three questions were rated as *satisfactory*. Six questions had results that were rated as *marginal* and three as *weak*.

The highest percentage of correct responses for a question on this topic was 81 per cent, achieved on question 12 (reproduced below). The interpretation panel considered this result *satisfactory*.

Question 12

Examine the following diagram:



An equation of the curve is:

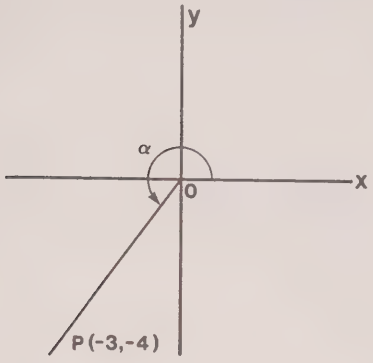
Student Responses

*81%	A)	$y = \sin x$
7%	B)	$y = \cos x$
1%	C)	$y = -\cos x$
9%	D)	$y = \frac{1}{2} \sin x$
1%		omit

Question 133 (reproduced below) tested students' knowledge of a fundamental definition. The result of 42 per cent received a *weak* rating.

Question 133

Examine the following diagram:



The value of cos α is

Student Responses

31%	A)	$\frac{4}{3}$
*42%	B)	$-\frac{3}{5}$
18%	C)	$-\frac{4}{5}$
6%	D)	$-\frac{5}{3}$
4%	omit	

Question 93 (reproduced below) and question 132 tested the knowledge of other basic concepts: periodicity and phase shift. The results of 37 per cent and 36 per cent correct were given ratings of *weak* and *marginal*, respectively.

Question 93

The period of $y = \frac{1}{3} \cos (2x)$ is:

Student Responses

30%	A)	$\frac{\pi}{3}$
24%	B)	$\frac{\pi}{2}$
*37%	C)	π
5%	D)	4
4%	omit	

Although the questions relating to the reciprocal functions were not well done, the interpretation panel suggested that they should not be given a higher priority since it is clear that the basic functions require more attention.

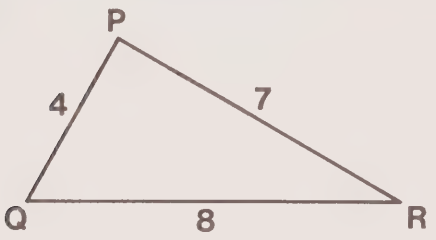
Applications of Trigonometric Functions

The interpretation panel rated the results as *marginal* on this topic. They expressed a concern about question 95 because of the potential confusion arising from contradictory information in the written portion and in the diagram. The results were rated as *strong* for two questions, *satisfactory* for four, and *marginal* for seven.

Questions 15, 94, and 96 all tested the ability to apply trigonometry to right triangles. The response to question 96 (75 per cent correct) was given a *strong* rating. The results for question 15 (69 per cent correct) were rated as *satisfactory*, and those for question 94 (61 per cent correct) were rated as *marginal*. Question 96 is reproduced below.

Question 96

Examine the following triangle:



The measure of $\angle Q$ is:

Student Responses

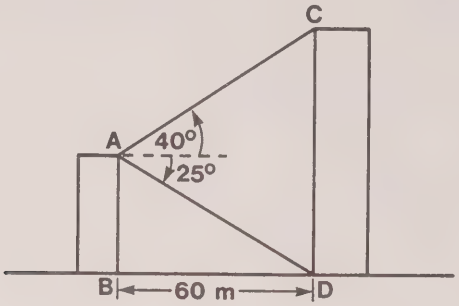
3%	A)	14°
15%	B)	30°
*75%	C)	61°
4%	D)	89°
3%	omit	

Question 16 (73 per cent correct) and question 52 (67 per cent correct) are standard questions on oblique triangles, and the results were rated as *satisfactory*.

Questions 51, 135, and 136 were more difficult applications. The result of 72 per cent correct for question 51 (reproduced below) was rated as *strong*. The results on the other two questions (40 and 52 per cent correct, respectively) were rated as *marginal*.

Question 51

Two buildings are 60 m apart. From the top of the smaller building, the angle of depression of the base of the taller building is 25° and the angle of elevation of the top is 40°.



The height of the taller building, to the nearest metre, is:

Student Responses

3%	A)	28 m
13%	B)	50 m
*72%	C)	78 m
8%	D)	129 m
4%	omit	

The interpretation panel rated the achievement of students on applications involving triangles as *satisfactory*.

Questions 17, 53, 97, and 137 tested students' ability with trigonometric identities and equations. With percentages of correct results ranging from 27 to 42, the rating for all these questions was *marginal*. Questions 17 and 97 are reproduced below.

Question 17

An expression equivalent to $\frac{\cos x - 1}{\cos x \sin x} + \frac{\sin x}{\cos x}$ is:

Student Responses

23%	A)	$\frac{(\cos x + 2)(\cos x - 1)}{\cos x \sin x}$
24%	B)	$\frac{1 + \cos x}{\sin x}$
18%	C)	$\frac{\cos x - \cos^2 x}{\sin x}$
*27%	D)	$\frac{1 - \cos x}{\sin x}$
8%	omit	

Question 97

If $0^\circ \leq \theta \leq 360^\circ$, then the exact values of θ which satisfy the equation $(\sin \theta)(2 \cos\theta - 1) = 0$ are:

Student Response

*30%	A)	$0^\circ, 60^\circ, 180^\circ, 300^\circ, 360^\circ$
29%	B)	$0^\circ, 60^\circ, 120^\circ, 180^\circ, 360^\circ$
19%	C)	$60^\circ, 300^\circ$
15%	D)	$0^\circ, 180^\circ, 240^\circ, 300^\circ, 360^\circ$
6%	omit	

The interpretation panel suggested that trigonometric identities do not require increased emphasis.

Exponential and Logarithmic Functions

Grade 12 (advanced level) mathematics students are expected to be able to graph exponential and logarithmic functions, to convert expressions from exponential to logarithmic form and from logarithmic to exponential form, and to solve problems involving these functions.

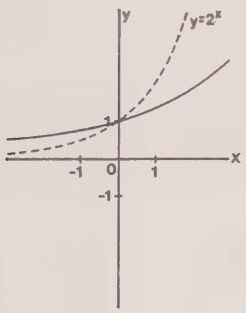
There were twelve questions on this topic. The interpretation panel rated student achievement on the topic as *marginal*. The rating of the results on four of the questions was *satisfactory*, on six *marginal*, and on two *weak*.

Question 99 (reproduced below) was the only question on the graphs of these functions. The results were rated as *marginal*.

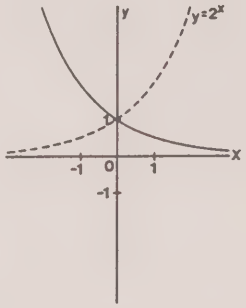
Question 99

In each of the following, the graph of $y = 2^x$ is the dotted curve. Which diagram also shows the graph $y = \left(\frac{1}{2}\right)^x$?

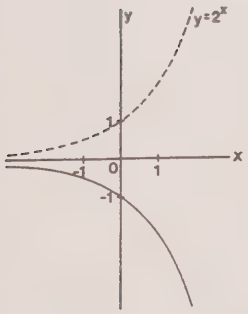
A)



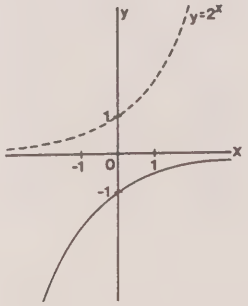
B)



C)



D)



Student Responses

34%	A)
*42%	B)
9%	C)
12%	D)
2%	omit

Students' understanding of exponents was tested in questions 18 and 57 (reproduced below). The results for question 57 were rated as *satisfactory*, but those for question 18 were rated as *weak*.

Question 18

The expression $3a^{\frac{1}{2}} \times 3a^{-\frac{1}{2}}$ equals:

Student Responses

36%	A)	1
*14%	B)	$9^{-\frac{1}{4}}$
23%	C)	$9a^{\frac{1}{4}}$
27%	D)	9a
1%	omit	

Question 57

If $10^a = 4$, then $10^{(1+2a)}$ is equal to:

Student Responses

16%	A)	26
19%	B)	40
*49%	C)	160
10%	D)	900
6%	omit	

Questions 19, 20, 58, 83, 138, and 139 tested students' understanding of logarithmic notation. The rating for the results was *satisfactory* for two of these questions, *marginal* for three, and *weak* for one. Question 83 is one of those for which the rating was *satisfactory*, question 138 is one for which the rating was *marginal*, and question 139 is the one for which the rating was *weak* (these three questions are reproduced below).

Question 83

If $\log_x 243 = 5$, then the value of x is:

Student Responses

4%	A)	1 215.0
18%	B)	48.6
*67%	C)	3.0
4%	D)	0.8
6%	omit	

Question 138

The expression $\log_2 64$ equals:

Student Responses

18%	A)	1.8
5%	B)	4.2
*52%	C)	6.0
21%	D)	8.0
4%	omit	

Question 139

If $\log N = n$, then $\log N^2$ is equal to:

Student Responses

5%	A)	$n + 2$
52%	B)	n^2
8%	C)	$\frac{n}{2}$
*30%	D)	$2n$
5%	omit	

Two of the three problems involving the exponential function had results that were rated as *marginal*. Question 59 (reproduced below) was one of these.

Question 59

Suppose the number of bacteria in a certain culture T hours from now will be $100\ (2.7)^T$. The time it will take for the bacteria count to reach 600 is approximately:

Student Responses

*54%	A)	2 h
21%	B)	3 h
15%	C)	6 h
5%	D)	12 h
5%	omit	

Graphs of Second-Degree Relations

The overall rating for the topic was *weak*. Of the nine questions, five had results that were rated as *weak*. The results on three were rated as *marginal*, and the results on one – question 22 (reproduced below) – as *satisfactory*.

Question 22

The transformation which maps the graph $x^2 - y^2 = 1$ to the graph of $\frac{x^2}{9} - \frac{y^2}{25} = 1$ is:

Student Responses

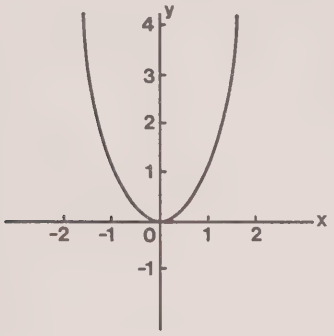
18%	A)	a stretch in the x-direction of factor $\frac{1}{3}$ and in the y-direction of factor $\frac{1}{5}$.
*50%	B)	a stretch in the x-direction of factor 3 and in the y-direction of factor 5.
10%	C)	a stretch in the x-direction of factor 9 and in the y-direction of factor 25.
18%	D)	a stretch in the x-direction of factor $\frac{1}{9}$ and in the y-direction of factor $\frac{1}{25}$.
3%	omit	

Question 21 (reproduced below) is one of those with a *marginal* rating.

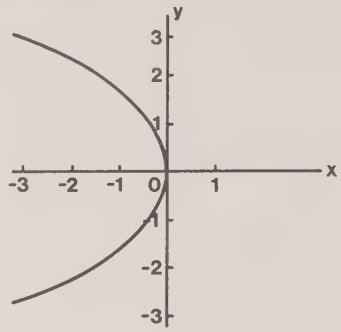
Question 21

Which of the following is the graph of $y = -\frac{1}{4}x^2$?

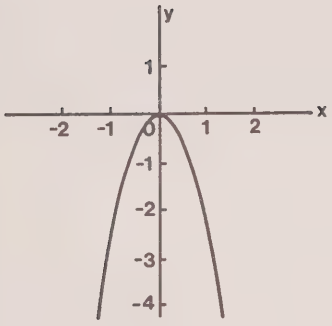
A)



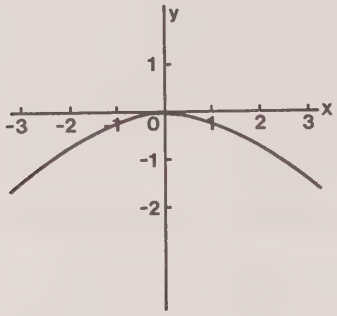
B)



C)



D)



Student Responses

3%	A)
9%	B)
29%	C)
*58%	D)
1%	omit

Question 144 (reproduced below) is one of those with a *weak* rating.

Question 144

State the length of the radius and the co-ordinates of the centre of the circle defined by:

$x^2 + (y + 5)^2 = 9$

Student Responses

Co-ordinates of centre		*Radius*	
37%	right	52%	right
37%	wrong	21%	wrong
26%	omit	27%	omit

*34% co-ordinates and radius correct

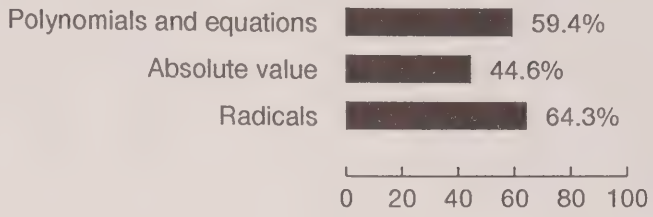
The interpretation panel recommended that the optional objective, Graphing Functions and Relations (d), be considered core and be used to give students increased understanding of the graphs of second-degree relations.

8.3 Algebraic Operations

Algebraic operations for Grade 12 (advanced level) mathematics include the factoring of quadratic polynomials and the sum and difference of cubes, division of a polynomial by a binomial, the use of the remainder theorem and the factor theorem, the solution of polynomial equations of degree two and greater, the definition of absolute value, the solution of equations and inequations involving absolute value, and the interpretation and evaluation of powers with integral bases and rational exponents.

The average percentage correct in algebraic operations was 57. The interpretation panel rated this student performance as *satisfactory*. The average percentage correct for each topic in algebraic operations is given in Figure 12. The panel ratings were *satisfactory* for polynomials and equations, *marginal* for absolute value, and *satisfactory* for radicals.

Figure 12
Student Achievement in Algebraic Operations by Topic



Polynomials and Equations

The overall rating for this topic was *satisfactory*. Of the ten questions, the results were rated as *strong* on one, *satisfactory* on three, and *marginal* on six.

Questions 1 and 43 tested divisions of polynomials and the remainder theorem. The performance on question 1 (reproduced below) was rated as *strong*. The results of question 43 (58 per cent correct) were rated as *satisfactory*.

Question 1

When $x^3 + 2x^2 + 5x + 11$ is divided by $(x + 2)$ the remainder is:

Student Responses

7%	A)	37
*82%	B)	1
6%	C)	0
3%	D)	−15
2%	omit	

Fifty-one per cent of students answered question 41 (reproduced below) correctly. The performance on this question testing knowledge of the factor theorem was rated as *marginal*.

Question 41

One of the factors of $x^3 - 7x^2 + 17x - 20$ is:

Student Responses

12%	A)	$x - 3$
10%	B)	$x + 3$
23%	C)	$x + 4$
*51%	D)	$x - 4$
4%	omit	

Three of the questions tested students' ability to solve equations. Question 121 tested the ability to solve a simple quadratic equation. The performance of 70 per cent correct was rated as *satisfactory*. With a more difficult quadratic equation, in question 42, the performance dropped to 56 per cent correct, which was rated as *marginal*. Also rated as *marginal* was the performance on question 2 (reproduced below), which tested students' ability to solve an equation of degree more than two.

Question 2

The roots of $2x^3 + 7x^2 = 2x + 7$ are:

Student Responses

8%	A)	$1, -\frac{1}{2}, -7$
10%	B)	$1, -1, -\frac{2}{7}$
*57%	C)	$1, -1, -\frac{7}{2}$
19%	C)	$1, -1, \frac{7}{2}$
6%	omit	

Of the three questions on factoring, question 103 (reproduced below) produced results which were rated as *satisfactory*.

Question 103

Factor completely $xy + 3x - 2y - 6$.

Student Responses

*70%	right
17%	wrong
13%	omit

Question 23 (reproduced below) and question 143 tested factoring of quadratic expressions. The results of 52 per cent and 46 per cent correct, respectively, were rated as *marginal.*

Question 23

Factor completely $6x^2 + 25x - 9$.

Student Responses

*52%	right
34%	incorrect
14%	omit

Although the interpretation panel identified factoring as an area of comparative weakness, the panel did not feel that the topic should receive more emphasis in Grade 12 (advanced level) mathematics.

Absolute Value

Of the four questions on this topic, the results on one were rated as *satisfactory,* on one as *marginal,* and on two as *weak.* The overall rating for the topic was *marginal.*

Student performance on question 56 (reproduced below) was rated as *satisfactory* and indicates that students understand the use of absolute-value notation in numerical cases.

Question 56

The expression $\frac{|-3| - (-3)}{2}$ equals:

Student Responses

1%	A)	–3
17%	B)	0
*72%	C)	3
10%	D)	$\frac{9}{2}$
1%	omit	

The *marginal* and *weak* ratings for performance on the other questions led the interpretation panel to conclude that students did not understand the basic concept of absolute value and that teachers needed to experiment with techniques for conveying this concept to students. Question 105 (reproduced below) is one of the questions for which performance was rated as *weak*.

Question 105

Solve $|x - 2| = 5$.

Student Responses

*38%	correct
46%	wrong
16%	omit

Radicals

There were four questions on this topic, and the overall rating was *satisfactory*. The panel rating was *satisfactory* for two questions and *marginal* for two questions.

Question 82 (reproduced below) and question 123 were those for which student performance was rated as *satisfactory*. These questions tested students' understanding of the concept of a radical.

Question 82

The expression $\sqrt{(-9)^2}$ is equal to:

Student Responses

23%	A)	-9
1%	B)	-3
7%	C)	$\|3\|$
*69%	D)	$\|-9\|$

Although 70 per cent of students answered question 63 (reproduced below) correctly, the interpretation panel rated the performance as only *marginal*, since the question tested a fundamental calculator skill for Grade 12 (advanced level) mathematics.

Question 63

Evaluate $17^{0.75}$ correct to 2 decimal places.

Student Responses

*70%	right
17%	wrong
13%	omit

8.4 Process Problems

The four questions of this type assessed students' ability to apply problem-solving strategies to non-routine problems. Students of Grade 12 (advanced level) mathematics are expected to be familiar with the following strategies: searching for a pattern, drawing a diagram, constructing a table, estimating, choosing mathematical operations and sequencing them, assuming a solution and working backwards, using a formula or writing an equation, solving a simpler problem, accounting for all possibilities, checking for hidden assumptions, making an

assumption, and drawing a conclusion. In solving a process problem, they should be able to select one of these strategies and use it to arrive at an answer. The strategies used by students to solve each problem are summarized in Table D-13 in Appendix D.

The overall rating for student achievement on process problems was *satisfactory*. The results on one question were rated as *superior*, on a second question as *satisfactory*, on a third question as *marginal*, and on the remaining question as *weak*. The interpretation panel noted that the questions for which the ratings were *satisfactory* or *strong* were ones which required mathematical maturity rather than a knowledge of mathematical content. The panel felt that problem 3, for which the rating was *weak*, was a poor question.

Problems 1 and 4 are reproduced below. Student performance on problem 1 of 83 per cent correct was rated as *superior*, and the performance of 25 per cent correct on problem 4 as *marginal*.

Problem 1

Tin Jee collects basketball cards. When they are stacked in piles of 2, there is 1 left over. When they are stacked in piles of 3, there is 1 left over. With piles of 5, there are none left over. If he has fewer than 50 basketball cards, how many cards does he have?

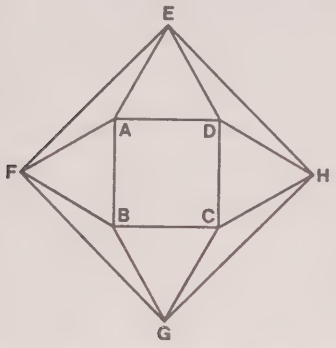
Student Responses

*83%	right
2%	right, except for a minor error
14%	wrong
1%	omit

For problem 1, the two most common strategies were searching for a pattern and estimating by guessing and checking, which were used by 29 and 25 per cent of students, respectively. Nine per cent of students used a formula or equation.

Problem 4

ABCD is a square with sides of length 6 cm. E, F, G, H are vertices of equilateral triangles constructed on the sides of the square. Calculate the area of square EFGH, correct to the nearest square centimetre.



Student Responses

*25%	right
14%	right, except for a minor error
56%	wrong
4%	omit

For problem 4, 40 per cent of students used equations involving the Theorem of Pythagoras or trigonometry. Thirteen per cent solved a similar problem and applied that solution to this problem.

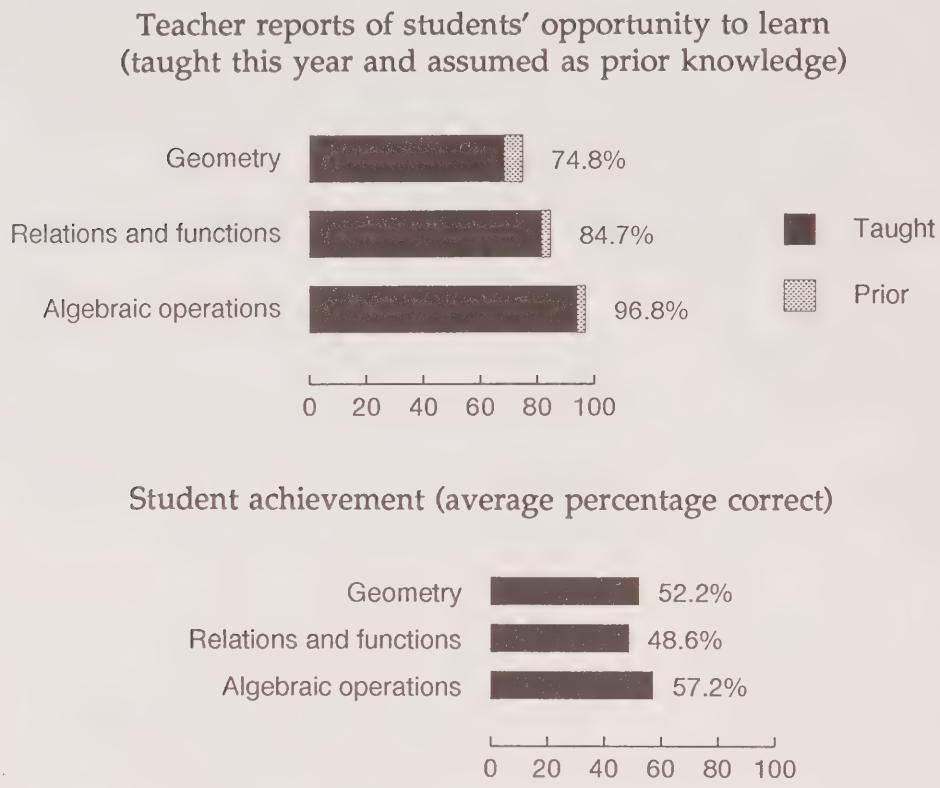
8.5 Opportunity to Learn

Teachers were asked to indicate whether or not they had taught the content required to answer each question in geometry, relations and functions, and algebraic operations. The results for each question are presented in Tables D-1 to D-11 in Appendix D. The summary data for each curriculum area are provided in Table D-15. Teachers who did not teach the content for a question were asked to indicate reasons for not teaching it. Results for each question are presented in Table D-16 in Appendix D.

The results for opportunity to learn and actual student achievement are summarized in the graphs in Figure 13. The graph for opportunity to learn shows the percentage of teachers who said that they taught the concepts or thought that the concepts had been taught in a prior year. These are the average percentages for all questions in each topic.

Students had a high opportunity to learn the concepts in algebraic operations and a somewhat lower opportunity in relations and functions. The opportunity to learn was approximately 75 per cent in geometry. The achievement levels do not reflect these differences in opportunity to learn. Opportunity-to-learn data were not obtained for process problems.

Figure 13
Summary of Results for
Opportunity to Learn (OTL) and Achieved Percentage Correct



9. Summary and Conclusions

This comprehensive review of the Grade 12 (advanced level) mathematics program in Ontario generated a wealth of information on student achievement, student attitudes, program planning, teaching resources and practices, and the context within which the program is delivered. A detailed examination of the information at the provincial, school board, and school levels will provide guidance on how to maintain existing strengths and how to generate improvement in areas where weaknesses were identified.

9.1 Student Achievement

Grade 12 (advanced level) students in Ontario achieved at levels that were rated as *satisfactory* or better for half of the curriculum areas. The strongest student performance was in the topic of similar figures in the geometry area.

Student performance in the following areas was judged to be *satisfactory*: congruence and parallelism, the circle, polynomials and equations, radicals, and process problems.

A *marginal* rating was given to student achievement in graphs of functions and relations, trigonometric functions, applications of trigonometric functions, exponential and logarithmic functions, and absolute value.

Teachers reported that students had the opportunity to learn the course content required to answer most of the questions in the Student Assessment Booklets. Opportunity to learn was highest for algebraic operations; it was also high for relations and functions, but somewhat lower for geometry.

9.2 Student Attitudes

Most students expressed positive attitudes towards mathematics. Seventy-eight per cent thought that mathematics was important for employment, 73 per cent said that mathematics helps them outside of school, and 93 per cent thought it was of equal importance for boys and girls. Fifty-one per cent of students felt that they were good in mathematics, and 33 per cent felt that mathematics was difficult. Sixty-one per cent of students liked mathematics. The mathematics topics that were liked by more than 50 per cent of students were congruence

and parallelism, trigonometric functions, factor theorem and solving polynomial equations, absolute value, and problem solving. When given a difficult problem, most students attempted a variety of approaches to reach a solution; few students gave up before reaching a solution.

9.3 Classroom Activities and Classroom Practices

The most commonly used methods of instruction were chalkboard or projector presentations to the whole class and having students work individually on assignments. These activities accounted for over 82 per cent of class time. Teacher demonstrations with concrete materials were used for an average of 8 per cent of class time.

Ninety-five per cent of teachers said that calculators were used in almost every class or during about half of the classes. Ninety-four per cent said that calculators were also used for tests. Fifty-four per cent of teachers reported that students did not have access to computers. When computers were available, teachers most frequently encouraged students to use them to practise new concepts, to learn new concepts and skills, and as an aid in doing projects.

Most students reported that they frequently worked on mathematics alone and that their teacher frequently taught mathematics to the whole class. Sixty-seven per cent of students never did projects in mathematics, and most students never used a computer in mathematics class. Fifty-six per cent of students had computers at home, but only 4 per cent used them for mathematics school work.

Seventy-three per cent of students did one or more hours of mathematics homework per week, 22 per cent did less than one hour, and 5 per cent did no mathematics homework.

It is hoped that this report will encourage discussion about mathematics in Ontario schools. Ontario educators should now take steps to:

- maintain the strengths that were identified by this review of the Grade 12 (advanced level) mathematics program;
- recommend and implement procedures that will lead to improvement in areas where weaknesses were identified.

Appendix A: Principal Questionnaire

The percentage of principals selecting each response for each question on the Principal Questionnaire is presented in this appendix. The percentages are based on the number of principals answering a specific question. The number of respondents is given for each question.

Many questions on this questionnaire asked for information about all three grade levels (Grades 8, 10, and 12). Only the data for Grade 12 are reported in this appendix. Questions 2 and 3, which apply to Grades 8 and 10, have been omitted.

If a large number of numerical responses was possible for a given question, these have been grouped into categories. The data for these questions represent the percentage of principals giving responses that fall in each category. The average of all responses is also given.

Question 1, concerning the number of students enrolled in the school, is an example of this type of question. The first category is 0-299 students. In the province, 5 per cent of principals said that there were fewer than 300 students enrolled in their schools; 33 per cent of principals said that there were 900-1199 students enrolled in their schools. The average number of students in all schools in the province is 991.

1. **How many students are enrolled in your school?**

No. of Students	*% of Principals*	
0-299	5%	Average
300-599	8%	number of
600-899	26%	students: 991
900-1199	33%	
1200-1499	18%	
1500 or more	10%	N = 92

Note: Question 2 applies to Grade 8 only, and question 3 applies to Grade 10 only.

4. **How many students are enrolled in the following in your school?**
(Include both semesters if applicable.)

Grade 12 Mathematics, Basic, General, and Advanced Levels

No. of Students	*% of Principals*	
0-34	3%	Average
35-49	0%	number of
50-99	14%	students: 164
100-149	32%	
150-199	24%	
200 or more	26%	N = 87

Grade 12 Mathematics, Advanced Level only

No. of Students	*% of Principals*	
0-19	3%	Average
20-29	2%	number of
30-49	9%	students: 104
50-69	16%	
70-99	28%	
100-149	16%	
150 or more	27%	N = 94

Percentage of Grade 12 mathematics students who are in the advanced level: 65.4%.

5. **What is the time allocation for mathematics instruction in your school?**

No. of Hours per Year	*% of Principals*	
0-109	1%	Average
110-114	49%	number of
115-119	35%	hours: 114
120 or more	15%	N = 94

6. **Which of the following best describes the community served by your school?**

Urban/suburban area	71%
Rural area	25%
Isolated area	4%

N = 97

7. Which of the following applies to the scheduling of your mathematics program?

Full year	23%	
Semestered	77%	
Other	0%	
Both	0%	N = 96

8. a) Is there an Immersion French or Extended French program in your school?

Yes	29%	
No	71%	N = 95

b) If Yes, is Grade 12 mathematics, advanced level, taught in French?

Yes	7%	
No	93%	N = 28

9. About how often do teachers in your school meet for the purpose of discussing the teaching of mathematics?

Once every week	33%	
Once every two or three weeks	23%	
Once every month or second month	40%	
Fewer than 5 times per year	3%	
Never	1%	
Only one teacher in the school	0%	N = 95

10. How much emphasis is placed on the following planning documents for mathematics?

	None	*Some*	*A Lot*	
Ministry of Education guidelines	0%	11%	89%	N = 95
School board documents	31%	31%	38%	N = 93
School-level planning documents/course of study	1%	9%	90%	N = 96

11. Does your school board have a procedure for regularly reviewing mathematics programs?

Yes	59%	
No	41%	N = 97

12. Does your school have a procedure for regularly reviewing mathematics programs?

Yes	88%	
No	12%	N = 97

13. When was the mathematics program last reviewed by either the school or the school board?

Has not occurred	7%	
Last school year (1988-89)	71%	
Two to four years ago	16%	
Five years or more	5%	
Currently	0%	N = 94

14. If a mathematics review was conducted, which of the following components did it include?

	Yes	*No*	
Review of planning documents	93%	7%	N = 85
Review of teaching practices	81%	19%	N = 86
Review of evaluation practices in schools	98%	2%	N = 88
Review of resources	81%	19%	N = 83
Administration of achievement test	41%	59%	N = 76

15. What changes occurred as a result of the last review of the mathematics program?

	Yes	*No*	
There was a change in:			
Teaching objectives	66%	34%	N = 87
Sequence and timing of programs	89%	11%	N = 85
Teaching strategies	70%	30%	N = 88
Student evaluation practices	72%	28%	N = 87
School resources	59%	41%	N = 83

16. Outside of regular professional development days, what activities or courses do teachers in your school engage in for the purpose of professional development for the teaching of mathematics?

	Yes	*No*	
Seminars/workshops	87%	13%	N = 94
Visiting other classrooms	52%	48%	N = 86
Conferences	92%	8%	N = 95
Peer coaching/job shadowing	38%	62%	N = 84
Credit courses	41%	59%	N = 86
Professional reading	97%	3%	N = 94

Appendix B: Teacher Questionnaire

The percentage of teachers selecting each response for each question on the Teacher Questionnaire is presented in this appendix. The percentages are based on the number of teachers answering a specific question. The number of respondents is given for each question.

If a large number of numerical responses was possible for a given question, these have been grouped into categories. The data for these questions represent the percentage of teachers giving responses that fall in each category. The average of all responses is also given. Question 19, which is reproduced below, is an example of this type of question.

---

19. What percentage of time is devoted to the problem-solving process?

% of Time	*% of Teachers*		
0-19%	28%	Average	
20-29%	29%	percentage	
30-39%	18%	of time:	30%
40-49%	8%		
50-59%	5%		
60% or more	13%		N = 175

In the province, 28 per cent of teachers said that they spent less than 20 per cent of their class time on the problem-solving process in mathematics; 13 per cent of teachers said that they spent 60 per cent or more of their class time on problem solving. The average percentage of time spent on problem solving by teachers in the province was 30 per cent.

Section 1: PERSONAL/PROFESSIONAL BACKGROUND

1. **Are you male or female?**

Male	69%
Female	31%

N = 173

2. **How many years' full-time teaching experience do you have?** ***(Include the current year and round to the nearest year any part-time experiences you have.)***

1 year	1%
2 to 5 years	10%
6 to 10 years	9%
11 or more years	80%

N = 175

3. **How many years have you spent teaching mathematics?** ***(Include the current year and round to the nearest year any part-time experiences you have.)***

	Elementary	*Secondary*
1 year	23%	1%
2 to 5 years	46%	12%
6 to 10 years	15%	10%
11 or more years	15%	77%
	N = 13	N = 175

4. **How many full-year courses in mathematics methods were included in your teacher education?**

No course in mathematics methods	8%
Some instruction as part of a general course	13%
One full course in mathematics methods	39%
Two or more full courses in mathematics methods	40%

N = 173

5. **How many full-year university courses do you have in mathematics?**

Zero	1%
One	1%
Two	2%
Three	5%
Four	5%
Five or more	86%

N = 175

6. **How many courses in measurement and evaluation or testing were included in your teacher training and/or graduate courses?**

Zero	38%
One	42%
Two	14%
Three or more	6%

N = 173

7. **Have you received training on the instructional use of computers with respect to the teaching of mathematics?**

Yes	41%
No	59%

N = 175

Section 2: TEACHING PRACTICES

8. What importance do you place on the instruction of the following topics?

	Not Important	*Important*	*Very Important*	
Congruence and parallelism	13%	62%	26%	N = 172
Similar figures	9%	65%	26%	N = 174
The circle	11%	60%	29%	N = 174
Graphs of functions and relations	1%	27%	72%	N = 174
Trigonometric functions	1%	13%	86%	N = 174
Applications of trigonometric functions	2%	21%	78%	N = 175
Exponential and logarithmic functions	5%	45%	50%	N = 175
Graphs of second-degree relations	2%	49%	48%	N = 174
Polynomials and equations	1%	22%	77%	N = 174
Absolute value	17%	67%	16%	N = 175
Radicals	19%	62%	19%	N = 175
Problem solving	1%	26%	74%	N = 175

9. How much time per week outside of class do you think that the average student in your class will take to complete his/her mathematics homework assignments?

Less than 1 hour	9%	
1 to 2 hours	24%	
Between 2 and 3 hours	39%	
3 hours or more	29%	N = 174

10. Do your students of mathematics have access to computers?

Yes	46%	
No (Go to Question 13)	54%	N = 173

11. Approximately what percentage of time do your mathematics students spend using computers per week?

% of Time	*% of Teachers*			
0-4% of time	57%	Average		
5-9% of time	22%	percentage		
10-14% of time	13%	of time:	4%	
15-19% of time	0%			
20% or more	7%			N = 67

12. Do you encourage your mathematics students to use computers to do any of the following?

	Yes	*No*	
To check answers to exercises	16%	84%	N = 75
To practise new concepts	41%	59%	N = 78
To learn new concepts and skills	40%	60%	N = 78
To take tests	0%	100%	N = 72
As an aid to do projects	39%	61%	N = 74

13. In your mathematics classes how often are calculators used?

Often (during every or almost every class)	75%	
Occasionally (during about half the lessons)	20%	
Rarely	5%	
Never	0%	N = 175

14. Do you encourage your mathematics students to use calculators to do any of the following?

	Yes	*No*	
To check answers to exercises	91%	9%	N = 172
To practise new concepts	75%	25%	N = 170
To learn new concepts and skills	75%	25%	N = 169
For use during tests	94%	6%	N = 174
As an aid to do projects	82%	18%	N = 165

15. How much emphasis is placed on each of the following in preparing lessons in mathematics?

	Not Available	*None*	*Some*	*A Lot*	
Ministry of Education guideline	0%	3%	39%	58%	N = 175
Board materials	19%	25%	43%	13%	N = 170
Textbook or teacher's edition of textbook	1%	1%	24%	74%	N = 174
Materials prepared by yourself or other teachers in your school	0%	0%	34%	66%	N = 175
Journals or other teacher reference books	7%	19%	66%	7%	N = 175
Commercially produced supplementary materials	16%	37%	46%	1%	N = 173
Materials from teacher education courses	9%	59%	30%	1%	N = 174
Materials from your in-service workshops, conferences, and seminars	9%	28%	61%	2%	N = 174
Suggestions from students	8%	23%	66%	3%	N = 173

16. What percentage of time during the school year do you use each of the following methods or aids for the teaching of mathematics? ***(The percentages should total 100%.)***

A chalkboard and/or projector presentation to the class

% of Time	*% of Teachers*			
0-24%	4%	Average		
25-49	25%	percentage		
50-74%	49%	of time:	57%	
75% or more	22%			N = 175

Teacher demonstration with concrete materials to the class

% of Time	*% of Teachers*			
0-4%	15%	Average		
5-9%	37%	percentage		
10-19%	41%	of time:	8%	
20-29%	6%			
30% or more	2%			N = 158

Students working cooperatively in groups

% of Time	*% of Teachers*			
0-4%	16%	Average		
5-9%	17%	percentage		
10-19%	33%	of time:	14%	
20-29%	21%			
30% or more	12%			N = 166

Students working individually

% of Time	*% of Teachers*			
0-9%	15%	Average		
10-19%	18%	percentage		
20-29%	24%	of time:	25%	
30-39%	24%			
40% or more	20%			N = 174

Note: The sum of the four percentages specified by teachers was often greater than 100%. Therefore, the sum of the four averages may be greater than 100%.

17. Which of the following materials do you have access to during the teaching of mathematics?

	Yes	*No*	
Blackline masters (photocopy masters)	72%	28%	N = 172
Commercially produced audio-visual materials	56%	44%	N = 172
Dot paper/grid paper	97%	3%	N = 174
Pattern blocks, geometric solids	63%	37%	N = 170
Geometry set	85%	15%	N = 173
Student-generated materials	34%	66%	N = 165
Textbooks	100%	0%	N = 174

18. How much emphasis do you place on each of the following in your mathematics class?

	None	*Some*	*A Lot*	
Blackline masters (photocopy masters)	24%	53%	23%	N = 173
Commercially produced audio-visual materials	59%	40%	1%	N = 173
Dot paper/grid paper	6%	72%	21%	N = 173
Pattern blocks and geometric solids	46%	53%	2%	N = 171
Geometry set	17%	73%	10%	N = 172
Student-generated materials	61%	36%	3%	N = 170
Textbooks	1%	14%	86%	N = 174

19. What percentage of time is devoted to the problem-solving process?

% of Time	*% of Teachers*			
0-19%	28%	Average		
20-29%	29%	percentage		
30-39%	18%	of time:	30%	
40-49%	8%			
50-59%	5%			
60% or more	13%			N = 175

20. Which of the following <u>best</u> describes the problem-solving component of your mathematics program?

I teach problem solving as a separate unit.	3%	
I integrate problem solving into the mathematics program throughout the year.	95%	
Both	2%	N = 173

21. What approaches do you use in teaching problem solving?

	Yes	*No*	
I provide students with many puzzles and problems to solve.	59%	41%	N = 168
I provide students with a systematic approach to solving problems (e.g. understand, plan, solve, look back).	96%	4%	N = 171
I teach specific strategies for solving problems (e.g. guess and check, make a list, draw diagram, etc.).	93%	7%	N = 174
I encourage students to develop their own strategies.	89%	11%	N = 171

22. How are the solutions to problems in mathematics dealt with by the students?

	Yes	*No*	
Students discuss their solutions with each other.	95%	5%	N = 172
Students report their results to the whole class.	69%	31%	N = 167
Students write up their solutions or the results of their discoveries and present them to the teacher.	73%	27%	N = 170

23. Do you use diagnostic pre-tests in your mathematics class?

Never	52%	
Sometimes	43%	
Often	5%	N = 175

24. On average, how often do you assess your students' learning of mathematics skills and concepts?

Less than once per month	0%	
Once per month	5%	
Twice per month	28%	
Three times per month	29%	
Four times per month or more	38%	N = 174

25. In mathematics, what per cent of the students' final mark is determined through the use of the following. ***(The total for the grade should add up to 100%.)***

	Percentage of Mark	*Percentage of Teachers*	*Average Percentage of Mark*	
Quizzes or short tests	0-9%	27%	11%	
	10-19%	61%		
	20-29%	10%		
	30% or more	2%		N = 157
Cumulative tests or end-of-unit tests	0-19%	1%	44%	
	20-29%	4%		
	30-39%	20%		
	40-49%	37%		
	50-59%	29%		
	60% or more	9%		N = 174
Formal examinations	0-29%	6%	38%	
	30-39%	31%		
	40-49%	49%		
	50-59%	14%		
	60% or more	1%		N = 175
Observation of student behaviour	0%	40%	5%	
	1-9%	29%		
	10-14%	27%		
	15% or more	3%		N = 131
Peer/self-evaluation	0%	84%	1%	
	1-4%	6%		
	5-9%	8%		
	10% or more	3%		N = 105
Others (projects, assignments, etc.)	0%	26%	6%	
	1-9%	34%		
	10-14%	31%		
	15% or more	9%		N = 134

Appendix C: Student Questionnaire

The percentage of students selecting each response is given for each question. The percentage of students omitting each question is also given. The percentages are based on the total number of students responding to the questionnaire in the province (4851).

1. Are you male or female?

52%	A) Male.
48%	B) Female.
0%	(omit)

2. In which year were you born?

4%	A) 1970 or before.
10%	B) 1971.
64%	C) 1972.
20%	D) 1973.
2%	E) 1974 or after.
0%	(omit)

3. What language do you most often speak at home?

86%	A) English.
1%	B) French.
13%	C) Other.
0%	(omit)

4. On average how much time out of school do you spend watching TV programs each day?

6%	A) None at all.
29%	B) Up to 1 hour.
37%	C) Between 1 and 2 hours.
20%	D) Between 2 and 3 hours.
8%	E) More than 3 hours.
0%	(omit)

5. About how much time do you spend on mathematics homework each week?

5%	A) I don't do any mathematics homework.
22%	B) Less than 1 hour.
36%	C) One to 2 hours.
37%	D) More than 2 hours.
0%	(omit)

6. How often do you get help with mathematics from someone in your family?

72%	A) Never or almost never.
24%	B) Occasionally.
3%	C) Often.
0%	(omit)

7. Which of the following describes how the mathematics program in your school is organized?

38%	A) Full-year.
61%	B) Semestered.
1%	C) Other.
0%	(omit)

DO YOU LIKE DOING MATHEMATICS QUESTIONS THAT DEAL WITH:

8. Congruence and Parallelism?

51%	A) Yes.
30%	B) No.
17%	C) I'm not sure.
3%	D) We do not do these kinds of questions.
0%	(omit)

9. Similar Figures?

49%	A) Yes.
26%	B) No.
20%	C) I'm not sure.
4%	D) We do not do these kinds of questions.
0%	(omit)

10. The Circle?

48%	A) Yes.
38%	B) No.
12%	C) I'm not sure.
3%	D) We do not do these kinds of questions.
0%	(omit)

11. Using Transformations?

46%	A) Yes.
35%	B) No.
16%	C) I'm not sure.
3%	D) We do not do these kinds of questions.
0%	(omit)

12. Trigonometric Functions?

55%	A) Yes.
28%	B) No.
13%	C) I'm not sure.
5%	D) We do not do these kinds of questions.
0%	(omit)

13. Applications of Trigonometry?

48%	A) Yes.
29%	B) No.
17%	C) I'm not sure.
6%	D) We do not do these kinds of questions.
0%	(omit)

14. Exponential and Logarithmic Functions?

38%	A) Yes.
26%	B) No.
22%	C) I'm not sure.
14%	D) We do not do these kinds of questions.
0%	(omit)

DO YOU LIKE DOING MATHEMATICS QUESTIONS THAT DEAL WITH:

15. Graphs of Second-Degree Relations?

26%	A) Yes.
33%	B) No.
29%	C) I'm not sure.
12%	D) We do not do these kinds of questions.
0%	(omit)

16. Factor Theorem and Solving Polynomial Equations?

72%	A) Yes.
20%	B) No.
8%	C) I'm not sure.
0%	D) We do not do these kinds of questions.
0%	(omit)

17. Absolute Value?

60%	A) Yes.
31%	B) No.
9%	C) I'm not sure.
0%	D) We do not do these kinds of questions.
0%	(omit)

18. Radicals?

47%	A) Yes.
39%	B) No.
13%	C) I'm not sure.
1%	D) We do not do these kinds of questions.
0%	(omit)

19. Problem Solving?

52%	A) Yes.
39%	B) No.
9%	C) I'm not sure.
0%	D) We do not do these kinds of questions.
0%	(omit)

The following questions ask about what you and your class do when you are learning mathematics. Think about what happens in your mathematics class.

20. How often do you do mathematics at the board or on the overhead projector?

27%	A) Almost every day.
11%	B) Once a week or so.
31%	C) Only once in a while.
31%	D) Never.
0%	(omit)

21. How often is mathematics presented to the whole class as a group?

91%	A) Almost every day.
2%	B) Once a week or so.
3%	C) Only once in a while.
3%	D) Never.
0%	(omit)

22. **How often do you do mathematics in small groups?**

9%	A) Almost every day.
8%	B) Once a week or so.
31%	C) Only once in a while.
52%	D) Never.
0%	(omit)

23. **How often do you do mathematics on your own in class?**

70%	A) Almost every day.
12%	B) Once a week or so.
15%	C) Only once in a while.
4%	D) Never.
0%	(omit)

24. **How often do you do projects on mathematics topics?**

1%	A) Almost every day.
3%	B) Once a week or so.
30%	C) Only once in a while.
67%	D) Never.
0%	(omit)

25. **How often do you do projects in other subjects that involve the use of mathematics?**

11%	A) Almost every day.
14%	B) Once a week or so.
53%	C) Only once in a while.
22%	D) Never.
0%	(omit)

26. **How often do you write tests or quizzes in mathematics?**

3%	A) Almost every day.
67%	B) Once a week or so.
29%	C) Only once in a while.
0%	D) Never.
0%	(omit)

27. **How often do you use a scientific calculator to do mathematics in class?**

77%	A) Almost every day.
13%	B) Once a week or so.
9%	C) Only once in a while.
2%	D) Never.
0%	(omit)

28. **How often do you use a computer to do mathematics in class?**

1%	A) Almost every day.
1%	B) Once a week or so.
11%	C) Only once in a while.
86%	D) Never.
0%	(omit)

29. Do you use a scientific calculator at home?

19%	A) No.
81%	B) Yes.
0%	(omit)

30. Do you have a computer at home?

44%	A) No.
13%	B) Yes, but I don't use it.
39%	C) Yes, I use it but not for mathematics schoolwork.
0%	D) Yes, I use it for mathematics schoolwork only.
4%	E) Yes, I use it for mathematics schoolwork and other things.
0%	(omit)

The following questions ask how you feel about the mathematics you are learning in school.

31. Do you think mathematics is:

5%	A) more for males than females?
1%	B) more for females than males?
93%	C) for everyone equally?
0%	(omit)

32. I like mathematics.

19%	A) Strongly agree.
42%	B) Agree.
18%	C) Not sure.
14%	D) Disagree.
6%	E) Strongly disagree.
0%	(omit)

33. Mathematics helps me outside of school.

22%	A) Strongly agree.
51%	B) Agree.
14%	C) Not sure.
11%	D) Disagree.
2%	E) Strongly disagree.
0%	(omit)

34. Learning mathematics is mostly memorizing.

7%	A) Strongly agree.
30%	B) Agree.
13%	C) Not sure.
41%	D) Disagree.
9%	E) Strongly disagree.
0%	(omit)

35. I am good at mathematics.

11%	A) Strongly agree.
40%	B) Agree.
25%	C) Not sure.
17%	D) Disagree.
6%	E) Strongly disagree.
0%	(omit)

36. It is important to know mathematics in order to get a good job.

36%	A) Strongly agree.
42%	B) Agree.
11%	C) Not sure.
9%	D) Disagree.
2%	E) Strongly disagree.
0%	(omit)

37. Mathematics is difficult for me.

9%	A) Strongly agree.
24%	B) Agree.
19%	C) Not sure.
36%	D) Disagree.
12%	E) Strongly disagree.
0%	(omit)

38. When I do problems in mathematics, I usually have trouble understanding what the sentences say.

5%	A) Strongly agree.
18%	B) Agree.
18%	C) Not sure.
46%	D) Disagree.
13%	E) Strongly disagree.
0%	(omit)

39. There are usually <u>different</u> ways to solve mathematical problems.

28%	A) Strongly agree.
62%	B) Agree.
8%	C) Not sure.
2%	D) Disagree.
0%	E) Strongly disagree.
0%	(omit)

WHEN YOU DON'T UNDERSTAND A PROBLEM IN MATHEMATICS DO YOU:

40. Try different ways to solve it on your own?

40%	A) Often.
54%	B) Sometimes.
5%	C) Never.
0%	(omit)

41. Ask another student for help?

38%	A) Often.
56%	B) Sometimes.
6%	C) Never.
0%	(omit)

42. Ask your teacher for help?

22%	A) Often.
67%	B) Sometimes.
11%	C) Never.
0%	(omit)

43. Wait and ask someone at home?

4%	A) Often.
23%	B) Sometimes.
73%	C) Never.
0%	(omit)

44. Just give up?

9%	A) Often.
65%	B) Sometimes.
25%	C) Never.
2%	(omit)

Appendix D: Mathematics Achievement

The results for all achievement items for Grade 12 (advanced level) mathematics are presented in this appendix.

1. Item Statistics for Student Assessment Booklets and Process Problems (D-1 to D-12).

Each table contains the results for the set of items dealing with one of the twelve topics, three in geometry, five in relations and functions, three in algebraic operations, and the category of process problems. Within each table, the items are grouped according to level of complexity (recognition/recall and comprehension, application, and analysis/proof). The items are identified by booklet number and item number. The set of questions for each topic precedes each table.

For multiple-choice questions, the key identifies the correct response, and the percentage of students selecting each response is given under columns *A, B, C,* and *D*. The percentage of students omitting each item is given under *O*. *OTL* represents the percentage of teachers who said that they taught the content covered by the question or that they felt it was taught in a previous year. *Cor* is the average percentage correct achieved by students. Some questions required students to write a response in the booklet, and most of these were marked correct or incorrect. A more detailed scoring scheme was used for some of the open-ended questions. The categories used in these scoring schemes and the algorithm for determining a correct response are outlined in Table D-14. For questions with two parts in the scoring guide, an answer had to meet the criteria for both parts in order to be rated as a correct response. For open-ended questions, data are provided only for *O, OTL,* and *Cor*.

For multiple-choice questions, the percentages are based on the total number of students answering each booklet. For the province, the numbers of students are as follows:

Booklet number:	1	2	3	4
Number of females:	585	546	528	563
Number of males:	559	583	592	560
Total number of students:	1162	1144	1141	1137

The total number is not equal to the sum of the numbers of females and males because some students did not specify gender.

For open-ended questions, a number of teachers did not record marks on the student answer sheets. The percentages reported for the item statistics are based on the total number of students for whom marks were recorded. This number of students is given below each table that contains data for open-ended questions. No data are reported for questions for which marks were recorded for fewer than 20 per cent of students who should have answered a given question. The number recorded under the *O* column is the percentage of students for whom teachers marked the no-response category. In Table D-12, the number of marked responses *(N)* is given for each question.

The conclusions of the interpretation panel are also provided. The acceptable level *(Acc)*, the desirable level *(Des)*, and the rating *(Rtg)* are given for each question. These terms have been discussed in "3.3 Interpretation of Results".

2. Strategies for Process Problems (D-13)

This table gives the percentage of students who used each strategy in obtaining solutions to process problems.

3. Scoring Criteria for Selected Open-Ended Questions (D-14)

The percentage of students in each category of the scoring guide is given for each question. The algorithm for determining a correct response is also given.

4. Summary of Achievement by Topic (D-15)

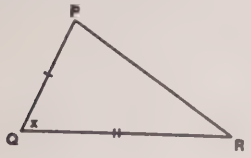
This table presents a summary of the data for groups of questions by topic. Averages are provided for percentage correct and opportunity to learn. Average percentage correct is given for males, females, and the total student population.

Panel ratings are given for each content area. The reported acceptable *(Acc)* and desirable *(Des)* levels were obtained by averaging the levels assigned to each question. The rating *(Rtg)* was assigned to each topic through deliberations of the panel. Panel ratings are not provided for the cognitive levels.

5. Opportunity to Learn (D-16)

Teachers were asked whether or not they had taught the content required to answer each question in the Student Assessment Booklets. Results for each question are presented in Table D-16. The percentage of teachers who said that they taught the content is reported in the *Taught* column, and the percentage of teachers who said that the content was not taught is reported in four columns according to the reason for not teaching it (assumed taught in a prior year, intended to teach it later in the year, content not in the curriculum, and other reasons).

4. Examine the triangle below.

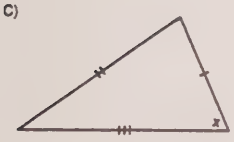


A triangle which is congruent to △PQR is:

A)

B)

C)



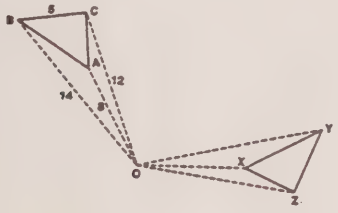
D)

9. The negation of the statement $\angle P > \angle Q$ is:

A) $\angle P \leq \angle Q$.
B) $\angle P < \angle Q$.
C) $\angle P = \angle Q$.
D) $\angle P \neq \angle Q$.

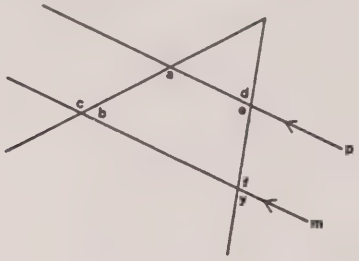
64. △XYZ is the image of △ABC under a clockwise rotation of 120° about O.

BC = 5 cm, OC = 12 cm, OA = 8 cm, OB = 14 cm.



The length of OZ is _____.
The measure of ∠AOX is _____.

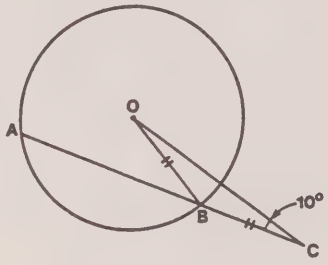
84. Examine the following diagram.



If line **p** is parallel to line **m**, then:

A) b = a
B) a + b = 180°
C) a + e = 180 °
D) b = f

128. In the diagram below, AB is any chord of the circle with centre O, OB = BC, and ∠OCB = 10°.



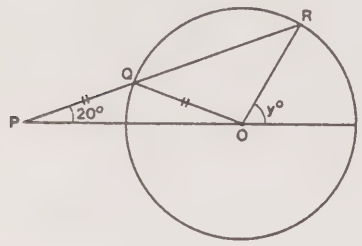
The measure of ∠AOB in degrees is:

A) 130°
B) 140°
C) 150°
D) 160°

104. What is the length of a side of a rhombus with diagonals of length 24 cm and 10 cm?

Answer: _____cm

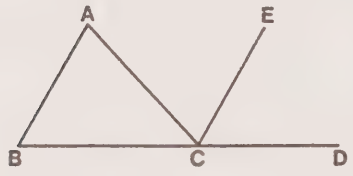
129. PQR is a secant to the circle below with centre O and PQ = OQ.



The value of y is:

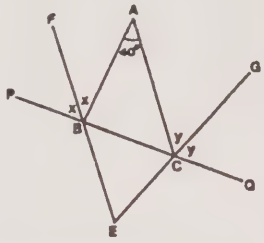
A) 40
B) 60
C) 80
D) 140

25. Given △ABC with BC produced to D. EC bisects ∠ACD and EC || AB.



Prove: △ABC is isosceles.

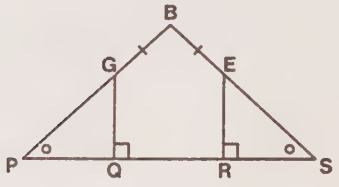
44. Given the following diagram,



the degree measure of ∠BEC is:

A) 40°
B) 70°
C) 80°
D) 110°

145. In the diagram below, GB = BE, ∠P = ∠S, GQ ⊥ PS and ER ⊥ PS.



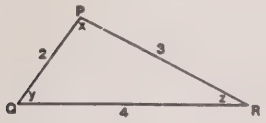
Prove: GQ = ER

Table D-1
Geometry: Congruence and Parallelism

Level	*Item*	*Key*	*A*	*B*	*C*	*D*	*E*	*O*	*OTL*	*Cor*	*Panel Rating* *Acc*	*Des*	*Rtg*
Recognition/Recall and Comprehension	1-4	(A)	86	1	7	5		0	95	86	80	90	4
	1-9	(A)	14	61	3	20		2	51	14	60	80	4
	2-64*							17	48	45	75	90	2
	3-84	(B)	10	79	4	6		0	92	79	80	90	4
	4-128	(B)	10	60	7	21		2	69	60	70	85	3
Application	3-104*							37	79	17	65	80	1
	4-129	(B)	14	67	12	5		3	65	67	70	85	3
Analysis/Proof	1-25*							24	89	31	65	80	2
	2-44	(B)	14	67	12	5		3	91	67	55	70	3
	4-145*							21	86	25	60	80	1

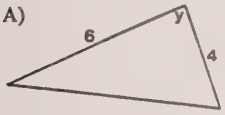
*Number of students for open-ended questions is 1137, 1130, 1151, and 1126 (order as in table).

45. Examine the triangle below.

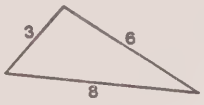


The following diagrams are not drawn to scale. Which triangle is similar to $\triangle PQR$:

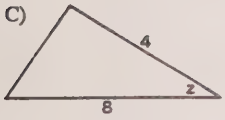
A)



B)

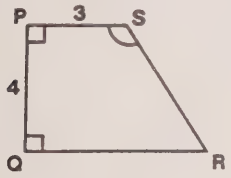


C)



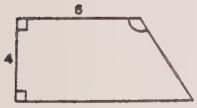
D)

87. Examine quadrilateral PQRS.

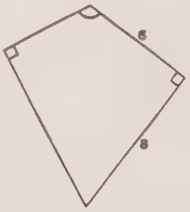


Which quadrilateral is similar to PQRS?

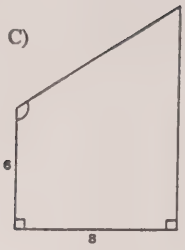
A)



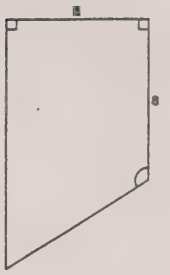
B)



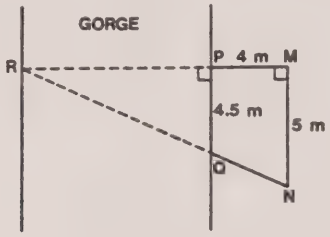
C)



D)



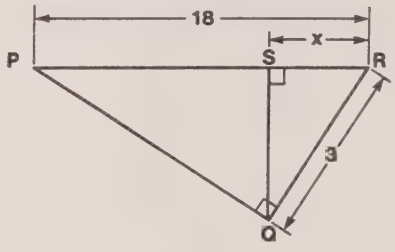
6. During a course in orienteering, you have to determine the distance across a gorge. **R** is a rock on one edge of the gorge. You position markers **P, Q, M** and **N** as shown in the diagram.



If **P** and **Q** are on the edge of the gorge, then the distance PR across the gorge is:

A) 3.2 m
B) 3.6 m
C) 36.0 m
D) 40.0 m

7. In the diagram below, PR = 18, QR = 3, and SR = x.



x is equal to:

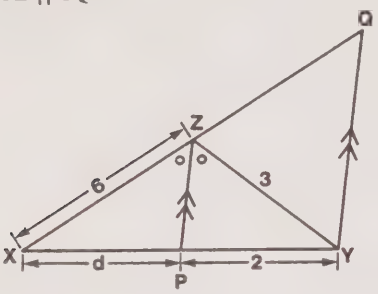
A) 2

B) $\frac{3}{2}$

C) 1

D) $\frac{1}{2}$

85. In the diagram below, PZ is the bisector of $\angle XZY$ and PZ || YQ.



d is equal to:

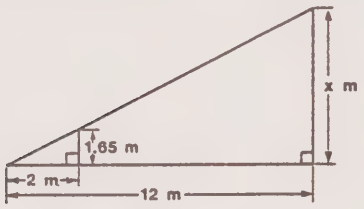
A) 1

B) $\frac{4}{3}$

C) 4

D) 9

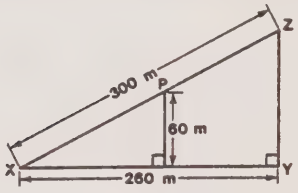
124. At a certain time of day a tree casts a shadow that is 12 m long. At the same time of day a man 1.65 m tall casts a shadow 2 m long.



The height of the tree is approximately:

A) 0.145 m
B) 9.9 m
C) 14.5 m
D) 990.0 m

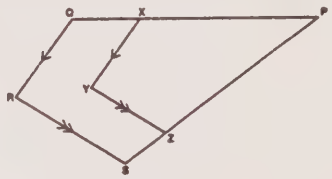
125. A ski hill 300 m long is modelled by the diagram below. At the top of the hill, the run is 260 m.



A skier stops at a point **P** on the hill at which the rise is 60 m. To reach the bottom of the hill, the skier still must ski a distance of approximately:

A) 130 m
B) 120 m
C) 69 m
D) 52 m

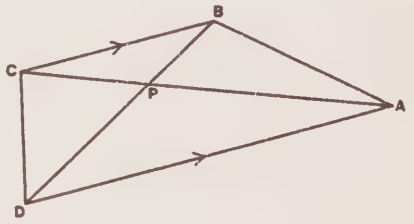
127. In the following diagram, quadrilateral PQRS is similar to quadrilateral PXYZ. The area of PQRS is 75 cm^2, and the area of PXYZ is 48 cm^2.



If PQ is 20 cm long, then the length of PX is:

A) 12.8 cm
B) 31.25 cm
C) 25 cm
D) 16 cm

65. Given the trapezoid below, ABCD, with AC and BD intersecting at **P**,



prove: ΔAPD is similar to ΔCPB.

Table D-2
Geometry: Similar Figures

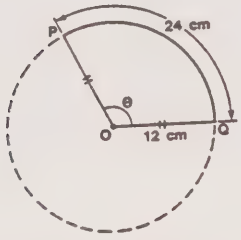
Level	*Item*	*Key*	*A*	*B*	*C*	*D*	*E*	*O*	*OTL*	*Cor*	*Panel Rating* *Acc*	*Des*	*Rtg*
Recognition/Recall and Comprehension	2-45	(D)	15	6	5	74		0	81	74	70	85	4
	3-87	(C)	4	1	92	3		0	77	92	80	90	5
Application	1-6	(C)	3	21	62	9		5	82	62	65	80	3
	1-7	(D)	32	19	9	36		4	77	36	65	80	1
	3-85	(C)	8	11	68	7		5	81	68	55	75	4
	4-124	(B)	1	86	11	1		1	84	86	80	90	4
	4-125	(B)	25	51	14	6		4	84	51	65	80	3
	4-127	(D)	49	5	2	40		4	64	40	60	70	2
Analysis/Proof	2-65*							18	80	63	70	85	3

*Number of students for question 2-65 is 1134.

5. The set of all points in a plane which are 10 units from a point **P** in the plane is:

A) a circle with centre **P** and radius 5 units.
B) a circle with centre **P** and radius 10 units.
C) a straight line which is 10 units from **P**.
D) two parallel lines, each of which is 10 units from **P**.

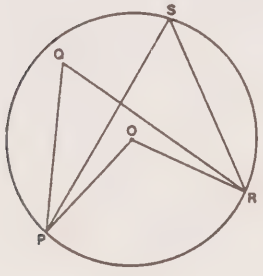
8. Examine the diagram below.



The measure of $\angle\theta$ in radians is:

A) $\frac{1}{2}$
B) 2
C) $\frac{2}{\pi}$
D) 2π

46. The circle below has centre **O**.



The inscribed angle(s) on arc PR shown in the diagram is (are):

A) $\angle$PSR only
B) $\angle$PQR only
C) $\angle$PSR and $\angle$PQR
D) $\angle$PSR, $\angle$PQR, and $\angle$POR

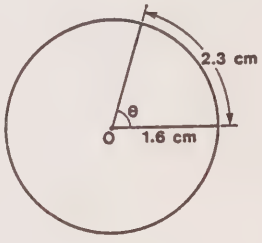
86. The measure of an angle in degrees is $\frac{180}{\pi}$. The measure of the angle in radians is:

A) $\frac{1}{\pi}$
B) 1
C) $\frac{180}{\pi^2}$
D) 57

89. The radius of a circle with centre **O** is 12 cm. The area of the sector with sector angle 1.3 radians to the nearest square centimetre is:

A) 187 cm^2
B) 94 cm^2
C) 55 cm^2
D) 16 cm^2

126. Examine the circle below with the centre **O**.



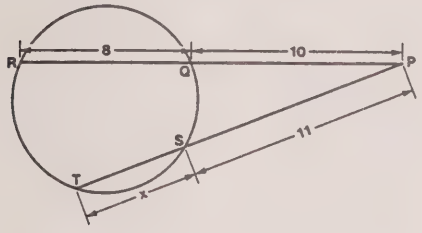
The measure of $\angle\theta$ to the nearest tenth of a degree is:

A) 1.4°
B) 41.2°
C) 82.4°
D) 103.0°

131. The measure of an angle in radians is $\frac{11}{3}\pi$. The measure of the angle in degrees is:

A) 1 320
B) 733
C) 660
D) 11.5

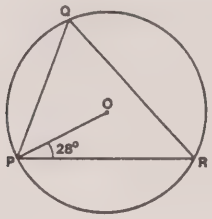
47. PQR and PST are secants to a circle.



The value of x correct to one decimal is:

A) 5.4
B) 7.0
C) 7.3
D) 8.8

88. O is the centre of the circle below.



If ∠OPR = 28°, then the measure of ∠PQR is:

A) 56°
B) 62°
C) 118°
D) 124°

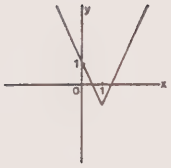
Table D-3
Geometry: The Circle

Level	*Item*	*Key*	*A*	*B*	*C*	*D*	*E*	*O*	*OTL*	*Cor*	*Panel Rating* *Acc*	*Des*	*Rtg*
Recognition/Recall and Comprehension	1-5	(B)	10	61	25	4		0	80	61	80	90	2
	1-8	(B)	10	30	39	17		5	77	30	70	85	1
	2-46	(A)	31	5	24	39		1	61	31	70	90	2
	3-86	(B)	19	47	10	23		2	95	47	70	90	3
	3-89	(B)	15	25	17	33		10	38	25	65	80	2
	4-126	(C)	12	14	69	2		4	65	69	70	85	3
	4-131	(C)	5	3	68	22		2	95	68	75	90	3
Application	2-47	(A)	37	18	16	25		4	45	37	60	75	3
	3-88	(B)	36	51	3	6		4	61	51	65	85	3

10. If $f(x) = 3 - x^2$ and $g(x) = \sqrt{6-x}$, then the value of $g(f(1))$ is:

A) 2
B) −2
C) $\sqrt{5}$
D) 5

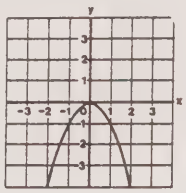
11. Which of the following is the graph of a function?

A) 
B) 
C) 
D) 

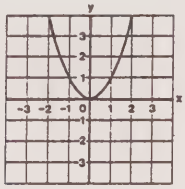
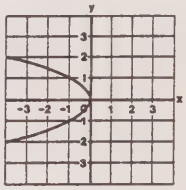
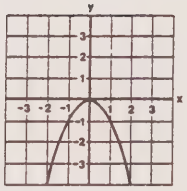
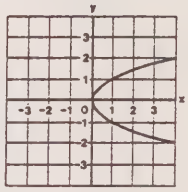
48. The graph of $y = f(x)$ is vertically stretched by factor 2, then horizontally shifted left by 3 units. Which of the following is the equation of the resulting graph?

A) $y = 2f(x - 3)$
B) $y = f(2x) - 3$
C) $y = f(\frac{1}{2}x + 3)$
D) $y = 2f(x + 3)$

55. Examine the graph below.



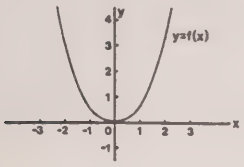
Which of the following is the graph of the **inverse** of the given relation?

A) 
B) 
C) 
D) 

90. The functions **f** and **g** are defined by $f(x) = x - 1$ and $g(x) = (x + 3)^2$. Therefore $g(f(x))$ is equal to:

A) $(x - 1)(x + 3)^2$
B) $(x + 3)^2 - 1$
C) $(2x - 2)^2$
D) $(x + 2)^2$

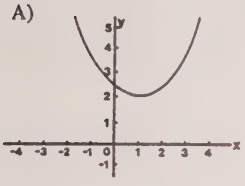
98. Examine the diagram below.



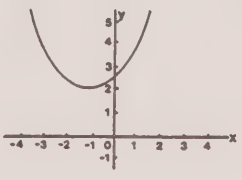
Which of the following is the graph of

$$y = \frac{1}{2}f(x - 1) + 2?$$

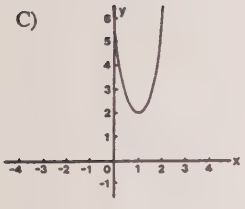
A)



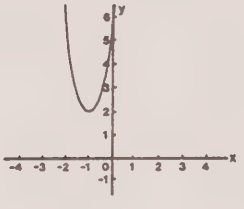
B)



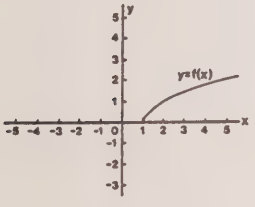
C)



D)

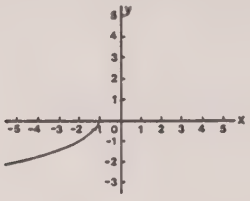


141. Examine the diagram below.

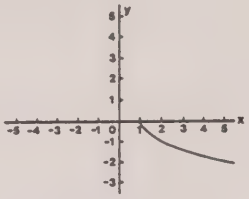


Which of the following is the graph of y = f(−x)?

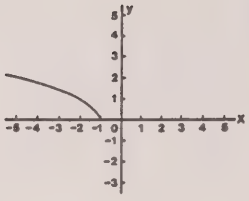
A)



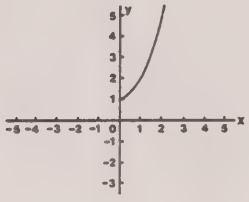
B)



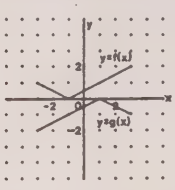
C)



D)

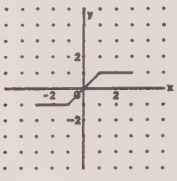


142. Examine the diagram below.

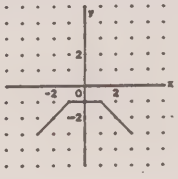


Which of the following is the graph of y = f(x) + g(x)?

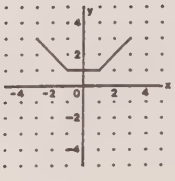
A)



B)



C)



D)

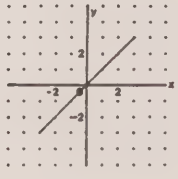
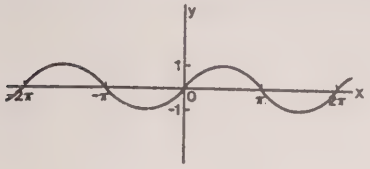


Table D-4
Relations and Functions: Graphs of Functions and Relations

Level	*Item*	*Key*	*A*	*B*	*C*	*D*	*E*	*O*	*OTL*	*Cor*	*Panel Rating* *Acc*	*Des*	*Rtg*
Recognition/Recall and Comprehension	1-10	(A)	76	6	11	4		2	96	76	70	85	4
	1-11	(C)	14	12	70	2		1	99	70	80	95	2
	2-48	(D)	15	8	13	64		0	95	64	70	85	3
	2-55	(B)	49	37	3	9		1	96	37	70	80	1
	3-90	(D)	30	21	3	46		1	96	46	75	85	2
	3-98	(A)	41	9	36	13		1	98	41	70	80	2
	4-141	(C)	14	36	36	12		2	93	36	70	85	1
	4-142	(A)	66	4	8	19		3	91	66	70	85	3

12. Examine the following diagram.



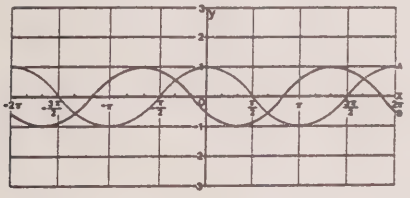
An equation of the curve is:

A) $y = \sin x$
B) $y = \cos x$
C) $y = -\cos x$
D) $y = \frac{1}{2}\sin x$

13. The range of $y = \sec \theta$ is:

A) $\{y \mid y \geq 1\}$
B) $\{y \mid y \leq -1\}$
C) $\{y \mid -1 \leq y \leq 1\}$
D) $\{y \mid y \geq 1 \text{ or } y \leq -1\}$

14. Examine the diagram below.



The transformation that maps graph A to graph B is:

A) a shift of $\frac{2\pi}{3}$ to the left.

B) a shift of $\frac{\pi}{3}$ to the right.

C) a shift of $\frac{\pi}{3}$ to the left.

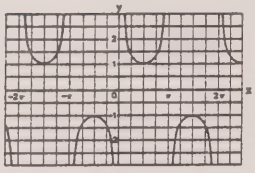
D) a shift of $\frac{2\pi}{3}$ to the right.

50. If $\sin \theta = \frac{1}{2}$, and $90° < \theta < 180°$, then the value of θ is:

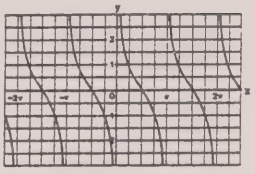
A) 150°
B) 120°
C) 30°
D) 0.0087°

54. The graph of $y = \csc x$, $-2\pi \leq x \leq 2\pi$, is:

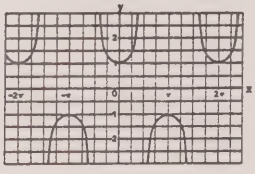
A)



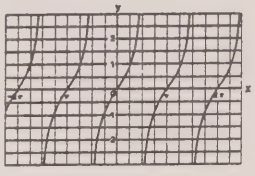
B)



C)



D)



91. Which of the following functions has a negative value?

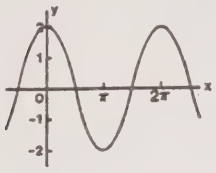
A) $\sin \frac{5\pi}{12}$

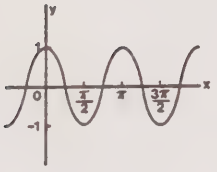
B) $\tan \frac{5\pi}{4}$

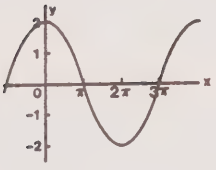
C) $\tan \frac{5\pi}{6}$

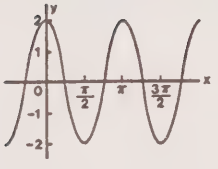
D) $\cos \frac{5\pi}{3}$

92. The graph of $y = 2\cos(2x)$ is:

A) 

B) 

C) 

D) 

93. The period of $y = \frac{1}{3}\cos(2x)$ is:

A) $\frac{\pi}{3}$

B) $\frac{\pi}{2}$

C) π

D) 4

132. The phase shift of $f(x) = 2\cos(3x - \pi)$ is:

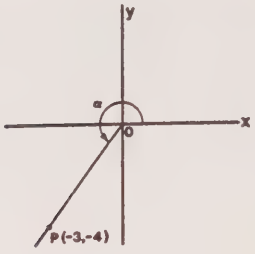
A) $-\pi$

B) $-\frac{\pi}{3}$

C) π

D) $\frac{\pi}{3}$

133. Examine the following diagram.



The value of $\cos\alpha$ is:

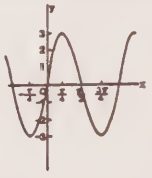
A) $\frac{4}{3}$

B) $-\frac{3}{5}$

C) $-\frac{4}{5}$

D) $-\frac{5}{3}$

134. Examine the following diagram.



An equation of the curve is:

A) $y = 3\sin(2x)$
B) $y = 3\cos(2x)$
C) $y = 3\sin(x)$

D) $y = 3\cos\left(\frac{x}{2}\right)$

49. The intercept of $y = \cos\left(x - \frac{x}{3}\right)$ with the y-axis is:

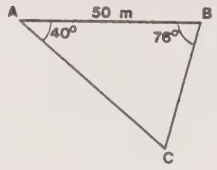
A) 1
B) 0.5
C) –0.5
D) –1

Table D-5
Relations and Functions: Trigonometric Functions

										Panel Rating		
Level	*Item*	*Key*	*A*	*B*	*C*	*D*	*E*	*O*	*OTL*	*Cor*	*Acc*	*Des*	*Rtg*
Recognition/Recall and Comprehension	1-12	(A)	81	7	1	9		1	95	81	80	95	3
	1-13	(D)	8	8	46	34		4	78	34	55	70	3
	1-14	(A)	52	11	18	17		2	94	52	65	80	3
	2-50	(A)	40	24	27	9		1	94	40	75	85	1
	2-54	(A)	40	11	28	18		3	78	40	65	80	2
	3-91	(C)	12	12	52	19		5	97	52	75	90	2
	3-92	(D)	20	6	21	51		2	95	51	70	85	2
	3-93	(C)	30	24	37	5		4	94	37	75	85	1
	4-132	(D)	12	26	23	36		3	90	36	65	85	2
	4-133	(B)	31	42	18	6		4	98	42	80	95	1
	4-134	(A)	42	11	32	12		3	94	42	70	85	2
Application	2-49	(B)	33	40	14	10		3	91	40	65	80	2

---

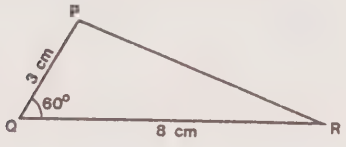
16. Examine the diagram below.



The length of AC to the nearest metre is:

A) 36 m
B) 46 m
C) 54 m
D) 76 m

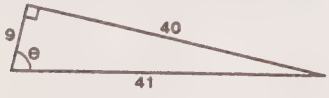
52. Examine the following triangle.



The length of PR to the nearest centimetre is:

A) 4 cm
B) 7 cm
C) 10 cm
D) 49 cm

94. Examine the triangle below.



Tan θ is equal to:

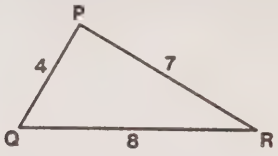
A) $\frac{9}{41}$

B) $\frac{9}{40}$

C) $\frac{40}{41}$

D) $\frac{40}{9}$

96. Examine the following triangle:



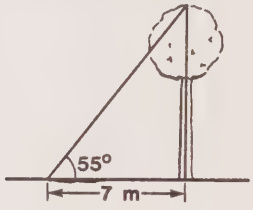
The measure of ∠Q is:

A) 14°
B) 30°
C) 61°
D) 89°

137. The number of roots of $2\cos x = 1$ in the domain $0 \le x \le 2\pi$ is:

A) 0
B) 1
C) 2
D) 4

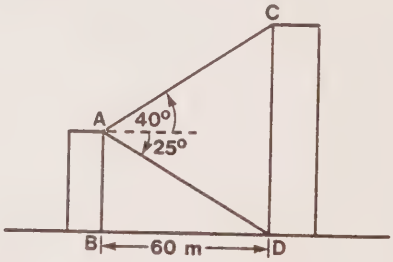
15. A tree casts a shadow 7 m long when the angle of elevation of the sun is 55°.



The height of the tree to the nearest tenth of a metre is:

A) 4.0 m
B) 4.9 m
C) 5.7 m
D) 10.0 m

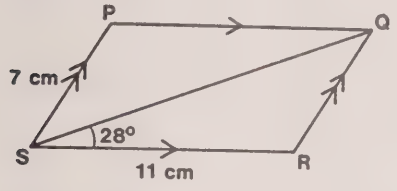
51. Two buildings are 60 m apart. From the top of the smaller building, the angle of depression of the base of the taller building is 25° and the angle of elevation of the top is 40°.



The height of the taller building, to the nearest metre is:

A) 28 m
B) 50 m
C) 78 m
D) 129 m

95. In parallelogram PQRS, SP = 7 cm, SR = 11 cm, ∠QSR = 28 °.



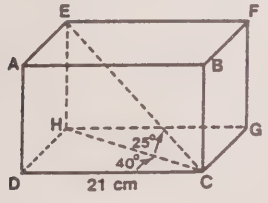
The measure of ∠SRQ to the nearest degree is:

A) 17°
B) 48°
C) 104°
D) 135°

97. If $0° \le \theta \le 360°$, then the exact values of θ which satisfy the equation $(\sin \theta)(2 \cos \theta - 1) = 0$ are:

A) 0°, 60°, 180°, 300°, 360°
B) 0°, 60°, 120°, 180°, 360°
C) 60°, 300°
D) 0°, 180°, 240°, 300°, 360°

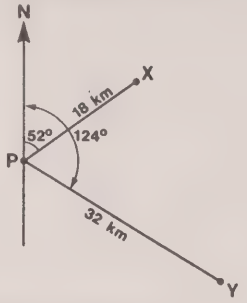
135. The diagonal of the base of a rectangular box, HC, makes and angle of 40° with the edge DC. The diagonal of the box, EC, makes an angle of 25° with the diagonal HC.



If the length of DC is 21 cm, then the length of EC to the nearest centimetre is:

A) 15 cm
B) 16 cm
C) 25 cm
D) 30 cm

136. Two ships sail from a port P at the same time. After three hours, their positions are X and Y as shown below.



The distance between X and Y to the nearest kilometre is:

A) 8 km
B) 31 km
C) 41 km
D) 992 km

17. An expression equivalent to $\frac{\cos x - 1}{\cos x \sin x} + \frac{\sin x}{\cos x}$ is:

A) $\frac{(\cos x + 2)(\cos x - 1)}{\cos x \sin x}$

B) $\frac{1 + \cos x}{\sin x}$

C) $\frac{\cos x - \cos^2 x}{\sin x}$

D) $\frac{1 - \cos x}{\sin x}$

53. An expression equivalent to $(1 + \sec x)(1 - \cos x)$ is:

A) $\tan x - \sin x$

B) $\frac{1 - \cos x}{\cos x}$

C) $\frac{\sin^2 x}{\cos x}$

D) $\cos x$

Table D-6
Relations and Functions: Applications of Trigonometric Functions

Level	*Item*	*Key*	*A*	*B*	*C*	*D*	*E*	*O*	*OTL*	*Cor*	*Panel Rating* *Acc*	*Des*	*Rtg*
Recognition/Recall and Comprehension	1-16	(C)	5	7	73	11		4	97	73	70	85	3
	2-52	(B)	5	67	17	7		3	96	67	70	90	3
	3-94	(D)	12	8	18	61		1	98	61	85	95	2
	3-96	(C)	3	15	75	4		3	96	75	70	85	4
	4-137	(C)	10	26	42	14		7	87	42	70	80	2
Application	1-15	(D)	4	6	17	69		2	98	69	80	90	3
	2-51	(C)	3	13	72	8		4	98	72	75	85	4
	3-95	(C)	5	20	50	20		5	93	50	70	80	3
	3-97	(A)	30	29	19	15		6	86	30	65	80	2
	4-135	(D)	7	9	39	40		5	87	40	65	80	2
	4-136	(B)	3	52	33	8		5	94	52	70	80	2
Analysis/Proof	1-17	(D)	23	24	18	27		8	87	27	55	70	2
	2-53	(C)	17	41	27	11		4	88	27	60	80	2

18. The expression $3a^{\frac{1}{2}} \times 3a^{-\frac{1}{2}}$ equals:

A) 1

B) 9

C) $9a^{-\frac{1}{4}}$

D) 9a

19. The expression $4^{\log_4 65}$ equals:

A) 65
B) 51.2
C) 12.34
D) 7.25

58. The equivalent form of $\log_m p = r$ is:

A) $p^r = m$
B) $r^m = p$
C) $p^m = r$
D) $m^r = p$

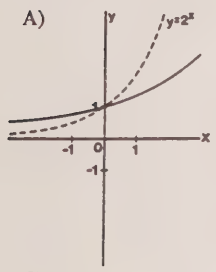
83. If $\log_x 243 = 5$, then the value of x is:

A) 1 215.0
B) 48.6
C) 3.0
D) 0.8

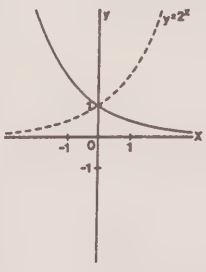
99. In each of the following, the graph of $y = 2^x$ is the dotted curve. Which diagram also shows the graph of

$y = \left(\frac{1}{2}\right)^x$?

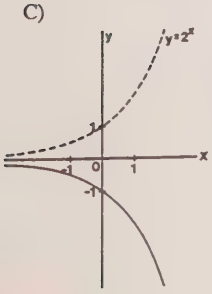
A)



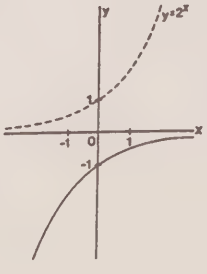
B)



C)



D)



138. The expression $\log_2 64$ equals:

A) 1.8
B) 4.2
C) 6.0
D) 8.0

139. If $\log N = n$, then $\log N^2$ is equal to:

A) $n + 2$

B) n^2

C) $\frac{n}{2}$

D) $2n$

59. Suppose the number of bacteria in a certain culture T hours from now will be $100(2.7)^T$. The time it will take for the bacteria count to reach 600 is approximately:

A) 2 h
B) 3 h
C) 6 h
D) 12 h

102. At the end of 8 years, the value of a $700 investment certificate is $1 034.22. If the interest is compounded annually, then the annual rate of interest for this certificate is:

A) 5 %
B) 6 %
C) 10 %
D) 105 %

122. The number of years it would take an investment of $1,000.00 to double in value at 9 %/a compounded annually is approximately:

A) 0.26
B) 0.34
C) 2.0
D) 8.0

20. If $\log_b 2 = \frac{1}{3}$, then $\log_b 32$ is equal to:

A) 5

B) $-\frac{3}{5}$

C) $\frac{5}{3}$

D) $\frac{3}{\log_2 32}$

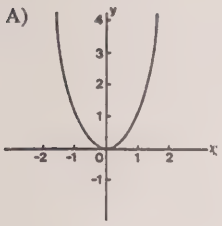
57. If $10^a = 4$, then $10^{(1+2a)}$ is equal to:

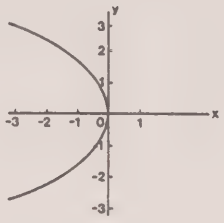
A) 26
B) 40
C) 160
D) 900

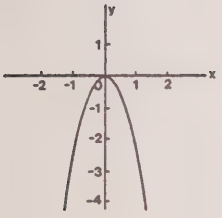
Table D-7
Relations and Functions: Exponential and Logarithmic Functions

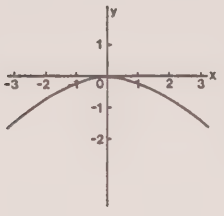
										Panel Rating		
Level	*Item*	*Key*	*A*	*B*	*C*	*D*	*E*	*O*	*OTL*	*Cor*	*Acc*	*Des*	*Rtg*
Recognition/Recall and Comprehension	1-18	(B)	36	14	23	27		1	96	14	75	90	1
	1-19	(A)	52	13	15	12		7	75	52	60	80	3
	2-58	(D)	12	17	12	54		5	78	54	75	90	2
	3-83	(C)	4	18	67	4		6	78	67	70	85	3
	3-99	(B)	34	42	9	12		2	80	42	70	80	2
	4-138	(C)	18	5	52	21		4	78	52	75	90	2
	4-139	(D)	5	52	8	30		5	75	30	75	90	1
Application	2-59	(A)	54	21	15	5		5	74	54	70	85	2
	3-102	(A)	41	38	12	4		5	63	41	65	85	2
	4-122	(D)	3	7	18	68		4	66	68	75	90	3
Analysis/Proof	1-20	(C)	18	10	43	20		9	75	43	60	75	2
	2-57	(C)	16	19	49	10		6	86	49	60	75	3

21. Which of the following is the graph of $y = -\frac{1}{4}x^2$?

A) 

B) 

C) 

D) 

22. The transformation which maps the graph $x^2 - y^2 = 1$ to the graph of $\frac{x^2}{9} - \frac{y^2}{25} = 1$ is:

A) a stretch in the x-direction of factor $\frac{1}{3}$ and in the y-direction of factor $\frac{1}{5}$.

B) a stretch in the x-direction of factor 3 and in the y-direction of factor 5.

C) a stretch in the x-direction of factor 9 and in the y-direction of factor 25.

D) a stretch in the x-direction of factor $\frac{1}{9}$ and in the y-direction of factor $\frac{1}{25}$.

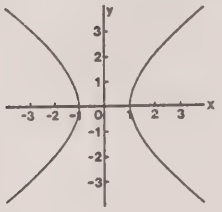
60. The centre and radius of the circle defined by $(x + 3)^2 + (y - 1)^2 = 4$ are:

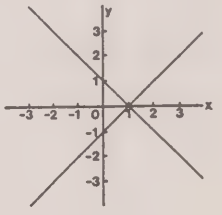
A) centre (3, −1); radius 2
B) centre (3, −1); radius 4
C) centre (−3, 1); radius 4
D) centre (−3, 1); radius 2

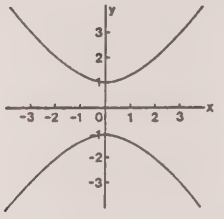
61. The coordinates of the vertices of the ellipse defined by $x^2 + 9y^2 = 81$ are:

A) (3, 0) and (−3, 0)
B) (0, 3) and (0, −3)
C) (9, 0) and (−9, 0)
D) (0, 9) and (0, −9)

62. Which of the following is the graph of $x^2 - y^2 = 1$?

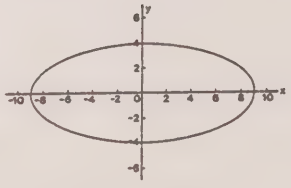
A) 

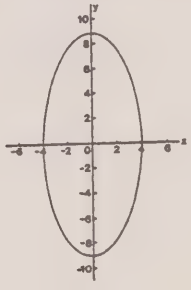
B) 

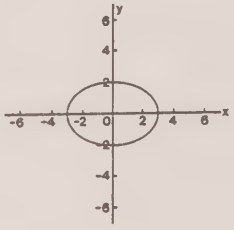
C) 

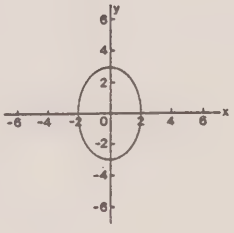
D) 

100. Which of the following is the graph of $\left(\frac{x}{2}\right)^2 + \left(\frac{y}{3}\right)^2 = 1$?

A) 

B) 

C) 

D) 

101. The transformation which maps the graph of $x^2 + y^2 = 9$ to the graph of $(x - 2)^2 + (y - 7)^2 = 9$ is:

A) a translation of 2 to the right and 7 up.
B) a translation of 2 to the left and 7 down.
C) a stretch in the x-direction of factor 2 and in the y-direction of factor 7.
D) a stretch in the x-direction of factor $\frac{1}{2}$ and in the y-direction of factor $\frac{1}{7}$.

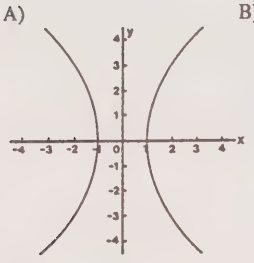
144. State the length of the radius and the co-ordinates of the centre of the circle defined by:

$x^2 + (y + 5)^2 = 9$

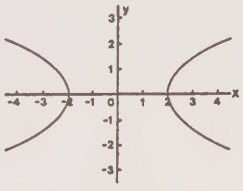
Centre: _____ Radius: _____

140. Which of the following is the graph of $x^2 - \frac{y^2}{4} = -1$?

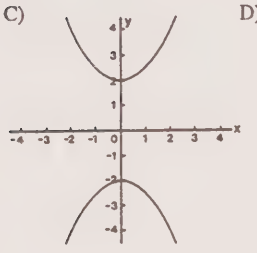
A)



B)



C)



D)

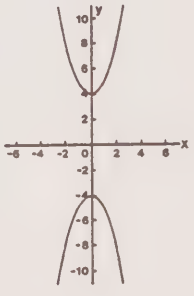


Table D-8
Relations and Functions: Graphs of Second-Degree Relations

										Panel Rating		
Level	*Item*	*Key*	*A*	*B*	*C*	*D*	*E*	*O*	*OTL*	*Cor*	*Acc*	*Des*	*Rtg*
Recognition/Recall and Comprehension	1-21	(D)	3	9	29	58		1	95	58	75	90	2
	1-22	(B)	18	50	10	18		3	42	50	60	75	3
	2-60	(D)	8	9	23	59		2	76	59	80	90	2
	2-61	(C)	13	33	33	17		4	57	33	75	85	1
	2-62	(A)	36	10	17	35		2	52	36	80	90	1
	3-100	(D)	11	31	15	38		4	59	38	75	85	1
	3-101	(A)	46	25	19	8		3	68	46	75	90	2
	4-140	(C)	24	10	35	26		5	51	35	70	85	1
	4-144*							25	81	34	75	90	1

*Number of students for question 4-144 is 1126.

Algebraic Operations: Polynomials and Equations

1. When $x^3 + 2x^2 + 5x + 11$ is divided by $(x + 2)$, the remainder is:

   A) 37
   B) 1
   C) 0
   D) −15

2. The roots of $2x^3 + 7x^2 = 2x + 7$ are:

   A) $1, -\frac{1}{2}, -7$

   B) $1, -1, -\frac{2}{7}$

   C) $1, -1, -\frac{7}{2}$

   D) $1, -1, \frac{7}{2}$

23. Factor completely $6x^2 + 25x - 9$.

    Answer: _____

41. One of the factors of $x^3 - 7x^2 + 17x - 20$ is:

    A) $x - 3$
    B) $x + 3$
    C) $x + 4$
    D) $x - 4$

42. The roots of $kx^2 - 3x + 4k = 0$ are:

    A) $\frac{3+\sqrt{9-16k}}{2}$ and $\frac{3-\sqrt{9-16k}}{2}$

    B) $\frac{3+\sqrt{9-16k^2}}{2k}$ and $\frac{3-\sqrt{9-16k^2}}{2k}$

    C) $\frac{3+\sqrt{9+16k^2}}{2k}$ and $\frac{3-\sqrt{9+16k^2}}{2k}$

    D) $\frac{3+\sqrt{9-16k^2}}{2}$ and $\frac{3-\sqrt{9-16k^2}}{2}$

43. When $2x^3 - 5x^2 + 6x - 1$ is divided by $x + 2$ the quotient is:

    A) $2x^2 - 9x + 15$
    B) $2x^2 - x + 4$
    C) $2x^2 - 9x + 24$
    D) $2x^2 - 9x - 12$

81. In factored form, $x^3 - 64$ is equivalent to:

    A) $(x - 4)(x^2 + 4x + 16)$
    B) $(x + 4)(x^2 - 4x + 16)$
    C) $(x - 4)(x + 4)^2$
    D) $(x - 4)^3$

103. Factor completely $xy + 3x - 2y - 6$.

     Answer: _____

121. The roots of $2x^2 - 11x + 5 = 0$ are:

     A) $\frac{1}{2}$ and 5

     B) $-\frac{1}{2}$ and −5

     C) $\frac{5}{2}$ and 1

     D) $-\frac{5}{2}$ and −1

143. Factor completely $4xy^2 - 12xy + 9x$.

     Answer: _____

Table D-9
Algebraic Operations: Polynomials and Equations

Level	*Item*	*Key*	*A*	*B*	*C*	*D*	*E*	*O*	*OTL*	*Cor*	*Panel Rating* *Acc*	*Des*	*Rtg*
Recognition/Recall and Comprehension	1-1	(B)	7	82	6	3		2	98	82	70	85	4
	1-2	(C)	8	10	57	19		6	98	57	70	80	2
	1-23*							14	99	52	80	90	2
	2-41	(D)	12	10	23	51		4	98	51	75	85	2
	2-42	(B)	20	56	15	6		3	99	56	70	85	2
	2-43	(C)	3	23	58	14		2	98	58	70	85	3
	3-81	(A)	53	16	10	21		1	98	53	70	80	2
	3-103*							13	99	70	75	85	3
	4-121	(A)	70	11	11	5		3	100	70	75	90	3
	4-143*							10	99	46	75	90	2

*Number of students for open-ended questions is 1151, 1132, and 1123 (order as in table).

Absolute Value

3. The expression $|x|$ may be defined as:

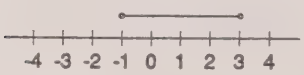
A) x, if $x = 0$
   $-x$, if $x > 0$
B) $\pm x$
C) x, if $x < 0$
   $-x$, if $x \geq 0$
D) x, if $x \geq 0$
   $-x$, if $x < 0$

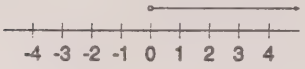
56. The expression $\frac{|-3| - (-3)}{2}$ equals:

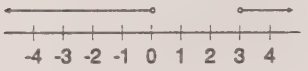
A) -3
B) 0
C) 3
D) $\frac{9}{2}$

105. Solve $|x - 2| = 5$.

130. The graph of $|x - 1| < 2$ is:

A)


B)


C)


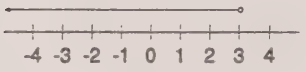
D)


Table D-10
Algebraic Operations: Absolute Value

Level	*Item*	*Key*	*A*	*B*	*C*	*D*	*E*	*O*	*OTL*	*Cor*	*Panel Rating* *Acc*	*Des*	*Rtg*
Recognition/Recall and Comprehension	1-3	(D)	9	50	8	32		1	96	32	65	85	2
	2-56	(C)	1	17	72	10		1	98	72	85	95	3
	3-105*							16	95	38	70	85	1
	4-130	(A)	36	7	26	28		2	91	36	65	85	1

---

*Number of students for question 3-105 is 1131.

24. Evaluate $(-15)^{\frac{1}{5}}$ correct to 2 decimal places.

Answer: _____

63. Evaluate $17^{0.75}$ correct to 2 decimal places.

Answer: _____

82. The expression $\sqrt{(-9)^2}$ is equal to:

A) -9
B) -3
C) $|3|$
D) $|-9|$

123. The expression $\sqrt[3]{(-2)^3}$ is equal to:

A) -2
B) -8
C) 2
D) $|-2|$

Table D-11
Algebraic Operations: Radicals

										Panel Rating		
Level	*Item*	*Key*	*A*	*B*	*C*	*D*	*E*	*O*	*OTL*	*Cor*	*Acc*	*Des*	*Rtg*
Recognition/Recall and Comprehension	1-24*							16	89	46	75	85	2
	2-63*							13	90	70	85	95	2
	3-82	(D)	23	1	7	69		0	98	69	75	85	3
	4-123	(A)	72	5	7	16		1	96	72	75	85	3

---

*Number of students for question 1-24 is 1152 and for question 2-63 is 1137.

Process Problems

Read the mathematics problems below carefully. Write your answer directly on this sheet. SHOW ALL YOUR WORK. Use the back of this page if you need more room.

1. Tin Jee collects basketball cards. When they are stacked in piles of 2, there is 1 left over. When they are stacked in piles of 3, there is 1 left over. With piles of 5, there are none left over. If he has fewer than 50 basketball cards, how many cards does he have?

2. On the beach during the summer Nina collected white and pink shells. One day she found 2 white shells and 1 pink shell. On the next day, she found 4 white shells and 4 pink shells, and on the third day she found 6 white shells and 7 pink shells. If this pattern continued, on what day did she find 30 more pink shells than white shells?

3. A triangle has sides of length 60 cm, 80 cm, and 100 cm. Determine the length of its shortest altitude.

4. ABCD is a square with sides of length 6 cm. E, F, G, H are vertices of equilateral triangles constructed on the sides of the square. Calculate the area of square EFGH, correct to the nearest square centimetre.

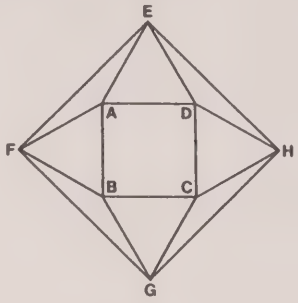


Table D-12
Process Problems

											Panel Rating		
Item	*Key*	*N*	*B*	*C*	*D*	*E*	*F*	*G*	*O*	*OTL**	*Cor*	*Acc*	*Des*	*Rtg*
1	B+C+D	1360	62	10	11	2	5	9	1		83	75	85	5
2	B+C+D	1244	51	3	2	21	13	9	1		56	65	85	3
3	B+C+D	1243	22	1	1	8	23	39	4		25	65	85	1
4	B+C+D	1210	24	0	0	14	21	35	4		25	55	75	2

B. Completes the solution correctly, using an appropriate strategy.

C. Gives the correct answer; starts an appropriate strategy, but does not complete it.

D. Gives the correct answer, but does not show reasoning/strategy.

E. Selects an appropriate strategy, but does not get correct answer due to a minor (e.g., computational) error.

F. Selects an appropriate strategy and shows understanding, but does not complete the solution or makes a major error in reasoning.

G. Attempts the question, but does not show understanding that would lead to a solution.

---

*Opportunity-to-learn data were not obtained for the process problems.

Table D-13
Percentage of Students Using Each Strategy for Process Problems

Item	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*I*	*S*
1	1	9	6	29	8	25	9	4	n/a	10
2	1	7	1	26	49	12	n/a	n/a	n/a	4
3	4	28	1	1	6	6	17	6	25	6
4	4	29	1	2	26	14	13	n/a	n/a	11

Strategies Represented by Each Letter

Item	*D*	*E*	*F*	*G*	*H*	*I*
1	Searching for a pattern	Constructing a table	Estimating; guess and check	Using a formula or equation	Accounting for all possibilities	n/a
2	Searching for a pattern	Constructing a table	Using a formula or equation	n/a	n/a	n/a
3	Estimating; guess and check	Drawing a scale diagram	Using similar triangles	Theorem of Pythagoras*	Area of a triangle*	Trigo-nometry*
4	Drawing an accurate diagram	Trigonometry*	Theorem of Pythagoras*	Solving a similar problem	n/a	n/a

Note: A - Did not attempt the question.

B - Attempted the question, but does not show understanding that would lead to a solution.

C - Answer only, right or wrong; no work shown.

S - Some other strategy.

*These strategies involved the use of a formula.

Table D-14
Scoring Criteria for Selected Open-Ended Questions

1-25	24%	A. Does not attempt the question.
	23%	B. Selects appropriate strategy and carries it through to produce a proof, using good form.
	8%	C. Selects appropriate strategy and carries it through to produce a proof, with error(s) in form.
	5%	D. Selects appropriate strategy, but makes a minor error.
	16%	E. Selects appropriate strategy, but makes a major logical error or does not complete the proof.
	24%	F. Attempts the question, but does not show understanding that would lead to a proof.
Correct: *B or C*		
2-64	45%	Student was required to get both length and angle correct.
4-144	34%	Student was required to get both centre and radius correct.
4-145	21%	A. Does not attempt the question.
	18%	B. Selects appropriate strategy and carries it through to produce a proof, using good form.
	7%	C. Selects appropriate strategy and carries it through to produce a proof, with error(s) in form.
	6%	D. Selects appropriate strategy, but makes a minor error.
	22%	E. Selects appropriate strategy, but makes a major logical error or does not complete the proof.
	26%	F. Attempts the question, but does not show understanding that would lead to a proof.
Correct: *B or C*		

The scoring criteria for the process problems were given in Table D-12. The remaining open-ended questions were simply scored correct or incorrect.

Table D-15
Summary of Achievement Data by Topic

Topic	No. of Items	Average Percentage Correct			OTL	Panel Rating		
		Female	Male	Total		Acc	Des	Rtg
Geometry	28	49.2	55.3	52.2	74.7	68	83	3
Congruence and parallelism	10	43.2	50.9	47.0	76.5	68	83	3
Similar figures	9	61.5	65.9	63.6	78.9	68	82	4
The circle	9	43.6	49.7	46.7	68.6	69	86	3
Recognition/recall and comprehension	14	52.2	59.4	55.8	73.1	-	-	-
Application and analysis/proof	14	46.2	51.2	48.6	76.3	-	-	-
Relations and Functions	54	47.1	50.3	48.6	84.7	71	85	2
Graphs of functions and relations	8	53.5	55.8	54.6	95.5	72	85	2
Trigonometric functions	12	44.3	47.1	45.6	91.5	70	85	2
Applications of trigonometric functions	13	51.1	54.5	52.7	92.7	70	83	2
Exponential and logarithmic functions	12	44.6	50.1	47.2	77.0	69	85	2
Graphs of second-degree relations	9	42.7	43.7	43.2	64.6	74	87	1
Recognition/recall and comprehension	40	48.0	50.2	49.1	84.7	-	-	-
Application and analysis/proof	14	44.5	50.4	47.3	84.7	-	-	-
Algebraic Operations	18	57.2	57.5	57.2	96.6	74	86	3
Polynomials and equations	10	60.2	58.9	59.4	98.6	73	86	3
Absolute value	4	44.0	45.5	44.6	95.0	71	88	2
Radicals	4	63.1	65.9	64.3	93.2	78	88	3
Process Problems	4	42.6	51.3	47.2	n/a	65	83	3

Table D-16
Teacher Responses for Opportunity to Learn

Percentage of Teachers

	Question Number	Taught	Reasons for Not Teaching			
			Taught prior	*Taught later*	*Not in curriculum*	*Other reasons*
Geometry	1-4	80	15	5	0	0
	1-5	69	11	14	2	4
	1-6	78	4	11	0	8
	1-7	77	1	15	1	8
	1-8	77	0	18	2	4
	1-9	41	10	4	21	25
	1-25	79	10	9	0	2
	2-44	76	15	6	1	2
	2-45	76	5	12	0	7
	2-46	59	2	24	2	14
	2-47	45	0	32	6	17
	2-64	25	23	10	21	20
	2-65	73	6	13	0	7
	3-84	76	16	7	0	1
	3-85	75	6	13	1	5
	3-86	95	0	5	0	1
	3-87	73	4	13	3	7
	3-88	59	2	29	2	8
	3-89	37	1	29	11	22
	3-104	59	20	8	3	10
	4-124	80	5	12	0	4
	4-125	81	2	12	0	5
	4-126	65	1	22	4	9
	4-127	60	4	18	8	11
	4-128	65	4	23	1	7
	4-129	61	4	27	1	7
	4-131	95	0	5	0	0
	4-145	79	7	10	1	2

Table D-16 (cont'd)
Teacher Responses for Opportunity to Learn

Percentage of Teachers

	Question Number	*Taught*	*Reasons for Not Teaching*			
			Taught prior	*Taught later*	*Not in curriculum*	*Other reasons*
Relations and Functions	1-10	91	5	1	1	1
	1-11	92	7	1	0	0
	1-12	95	0	5	0	0
	1-13	78	0	9	2	11
	1-14	94	0	6	0	1
	1-15	97	1	2	0	0
	1-16	97	0	3	0	0
	1-17	87	0	8	1	4
	1-18	92	4	3	1	0
	1-19	75	0	22	0	3
	1-20	75	0	22	0	3
	1-21	85	9	5	0	0
	1-22	40	1	35	4	20
	2-48	87	8	2	1	3
	2-49	90	1	6	1	2
	2-50	94	0	5	0	2
	2-51	98	1	2	0	0
	2-52	96	0	4	0	0
	2-53	88	0	7	1	4
	2-54	78	0	11	1	11
	2-55	91	5	2	1	1
	2-57	82	4	11	1	2
	2-58	78	0	21	0	2
	2-59	74	1	19	1	6
	2-60	63	14	17	1	5
	2-61	57	0	31	1	11
	2-62	52	0	36	1	11
	3-83	78	0	21	0	1
	3-90	89	7	1	1	1
	3-91	97	0	3	0	0
	3-92	95	0	5	0	1
	3-93	94	0	5	0	1
	3-94	98	1	2	0	0
	3-95	89	4	6	1	0
	3-96	96	0	4	0	0
	3-97	86	0	8	1	4
	3-98	86	12	1	0	1
	3-99	79	1	19	1	1

Table D-16 (cont'd)
Teacher Responses for Opportunity to Learn

Percentage of Teachers

			Reasons for Not Teaching			
	Question Number	*Taught*	*Taught prior*	*Taught later*	*Not in curriculum*	*Other reasons*
Relations and Functions (continued)	3-100	59	0	31	2	9
	3-101	61	7	24	2	6
	3-102	42	22	15	5	17
	4-122	46	20	12	6	16
	4-132	90	0	6	0	4
	4-133	98	0	2	0	0
	4-134	94	0	6	0	0
	4-135	84	2	3	6	5
	4-136	94	0	4	2	1
	4-137	87	0	7	1	5
	4-138	78	0	20	0	2
	4-139	75	0	23	0	2
	4-140	51	0	36	2	11
	4-141	82	11	2	0	5
	4-142	84	8	2	2	5
	4-144	65	16	14	1	4

Table D-16 (cont'd)
Teacher Responses for Opportunity to Learn

Percentage of Teachers

			Reasons for Not Teaching			
	Question Number	*Taught*	*Taught prior*	*Taught later*	*Not in curriculum*	*Other reasons*
Algebraic Operations	1-1	98	1	2	0	0
	1-2	97	1	2	0	0
	1-3	94	2	4	0	1
	1-23	98	2	1	0	0
	1-24	88	1	7	1	4
	2-41	98	0	2	0	0
	2-42	91	8	0	1	0
	2-43	97	1	2	0	0
	2-56	93	5	2	0	1
	2-63	89	1	9	0	2
	3-81	98	1	2	0	0
	3-82	89	8	2	0	1
	3-103	95	5	0	1	0
	3-105	93	2	4	1	1
	4-121	97	3	0	0	0
	4-123	92	5	4	0	0
	4-130	88	3	4	1	4
	4-143	95	4	0	1	0

Appendix E: Confidence Intervals for Achievement Results

This review used a matrix sampling of students and test questions. Questions were divided into four assessment booklets, and each student participating in the review responded to only one booklet. Since each student did not answer all questions, there is a degree of uncertainty associated with the achievement results reported in Appendix D. The estimates from matrix samples are generally good for scores that sum across the matrix samples – that is, that include questions from each booklet and therefore include information about all students. The statistical accuracy of an estimated response rate for a single item is, on the other hand, relatively low, since the item is answered by only one-quarter of the students.

The magnitude of the uncertainty for each curriculum area and for individual items is given in Table E-1 in the form of the 95 per cent confidence-band widths. For example, the band width for *Geometry* is ± 0.6 per cent and the average percentage correct as reported in Table D-15 is 52.2. The confidence interval for this topic is 52.2 per cent plus or minus 0.6 per cent, or 51.6 to 52.8 per cent. The average percentage correct would fall within this range 95 times out of 100 if the review were repeated with similar sampling procedures.

The band width for an individual item depends on the percentage correct. For example, an item answered correctly by 53 per cent of students has a band width of ± 2.9 per cent. The confidence interval for the item is 53 per cent plus or minus 2.9 per cent.

The confidence-band widths for each curriculum area for the averages for female and male students are given in Table E-2. Results for individual questions were not reported by gender.

Table E-1
Confidence-Band Widths for Achievement Topics and Individual Items

Confidence-band Width (±%)	*Topic*
0.6	*Geometry*
0.9	Congruence and parallelism
0.9	Similar figures
0.9	The circle
0.8	Recognition/recall and comprehension
0.8	Application and analysis/ proof
0.5	*Relations and Functions*
1.0	Graphs of functions and relations
0.9	Trigonometric functions
0.8	Applications of trigonometric functions
0.9	Exponential and logarithmic functions
1.0	Graphs of second-degree relations
0.6	Recognition/recall and comprehension
0.8	Application and analysis/ proof
0.8	*Algebraic Operations*
1.0	Polynomials and equations
1.3	Absolute value
1.3	Radicals
1.0	*Process Problems*

Confidence-band Width (±%)	*Individual Item*
1.7	Individual item with 10% correct response
2.3	Individual item with 20% correct response
2.7	Individual item with 30% correct response
2.9	Individual item with 40% correct response
2.9	Individual item with 50% correct response
2.9	Individual item with 60% correct response
2.7	Individual item with 70% correct response
2.3	Individual item with 80% correct response
1.7	Individual item with 90% correct response

Table E-2
Confidence-Band Widths for Achievement Topics by Gender

Confidence-band Width (±%)		*Topic*
Females	*Males*	
0.9	0.9	*Geometry*
1.3	1.2	Congruence and parallelism
1.3	1.2	Similar figures
1.3	1.3	The circle
1.1	1.0	Recognition/recall and comprehension
1.1	1.1	Application and analysis/proof
0.7	0.8	*Relations and Functions*
1.5	1.4	Graphs of functions and relations
1.2	1.3	Trigonometric functions
1.2	1.2	Applications of trigonometric functions
1.3	1.3	Exponential and logarithmic functions
1.4	1.4	Graphs of second-degree relations
0.8	0.8	Recognition/recall and comprehension
1.1	1.2	Application and analysis/proof
1.1	1.1	*Algebraic Operations*
1.4	1.3	Polynomials and equations
1.9	1.9	Absolute value
1.9	1.8	Radicals
1.5	1.4	*Process Problems*

Appendix F: Analysis of Planning Documents

Type of Document

1. Number of school-level documents: 96

2. Number of board-level documents: 19

The numbers in this appendix specify the percentage of documents that met specified criteria. One set of percentages is given for the 96 school-level documents that were submitted, and a second set of percentages is given for the 19 board-level documents.

	School		Board	
Course Guideline	**Yes %**	**No**	**Yes %**	**No**
3. Was the appropriate guideline identified (*Curriculum Guideline Mathematics Intermediate and Senior Divisions, 1985, Part 3: Grades 11 and 12, Advanced Level*)?	84	16	79	21
4. Did the document include a general description of the course?	44	56	63	37
5. Was the level of difficulty at which the course is offered indicated **or** the common course code identified?	98	2	100	0
6. Was a credit value indicated?	82	18	79	21

Suggested Minimum Number of Hours in Guideline		Objectives Listed			
		School		Board	
Content Area		Yes %	No	Yes %	No
7. Were the following content objectives listed and did they state a time requirement consistent with the guidelines indicated in the brochure?	*Suggested Hours*				
Geometry:					
• Congruence and parallelism	(8)	96	4	100	0
• Similar figures	(7)	96	4	100	0
• The circle	(8)	95	5	100	0
Relations and Functions:					
• Graphing functions and relations	(6)	97	3	100	0
• Trigonometric functions	(9)	99	1	100	0
• Applications of trigonometric functions	(11)	99	1	100	0
• Exponential and logarithmic functions	(10)	99	1	100	0
• Graphs of second-degree relations	(8)	99	1	100	0
Algebraic Operations:					
• Polynomials and equations	(7)	99	1	100	0
• Absolute value	(3)	96	4	95	5
• Radicals	(3)	91	9	90	10
Statistics:					
• Dispersion	(optional)	23	77	53	47
• Correlation	(optional)	23	77	47	53
Core extension or options	(30)	22	78	42	58
Minimum total time requirement	110				

Content Area		Minimum Time Requirement Met			
		School		Board	
		Yes %	No	Yes %	No
7. Were the following content objectives listed and did they state a time requirement consistent with the guidelines indicated in the brochure?	*Suggested Hours*				
Geometry:					
• Congruence and parallelism	(8)	86	14	94	6
• Similar figures	(7)	84	16	94	6
• The circle	(8)	90	10	100	0
Relations and Functions:					
• Graphing functions and relations	(6)	94	6	94	6
• Trigonometric functions	(9)	88	12	100	0
• Applications of trigonometric functions	(11)	84	16	94	6
• Exponential and logarithmic functions	(10)	88	12	100	0
• Graphs of second-degree relations	(8)	92	8	100	0
Algebraic Operations:					
• Polynomials and equations	(7)	99	1	100	0
• Absolute value	(3)	83	17	100	0
• Radicals	(3)	84	16	73	27
Statistics:					
• Dispersion	(optional)	4	96	0	100
• Correlation	(optional)	4	96	0	100
Minimum total time requirement	110				
8. Did the document address the following process components?					
• Proof		23	77	53	77
• Problem solving		29	71	50	50
9. Was there an indication of a scope and sequence **or** program delivery chart?		96	4	100	0

Course Objectives	School Yes %	School No	Board Yes %	Board No
10. Was a rationale/statement of philosophy outlined?	51	49	79	21
11. Were general course aims included?	74	26	84	16
12. Were objectives included for each content area?	73	27	84	16
13. Were the following types of objectives included?				
(a) Attitudes	32	67	21	79
(b) Skills	75	25	90	10
(c) Knowledge	85	15	100	0
14. Were the objectives mainly stated in terms of students' learning outcomes for the following?				
(a) Attitudes	18	82	21	79
(b) Skills	55	45	68	32
(c) Knowledge	57	43	68	32

Teaching Methods	School Yes %	School No	Board Yes %	Board No
15. Did the documents include a multi-resource approach to mathematics?	25	75	47	53
16. Were possible instructional methods/ teaching strategies described?	40	60	68	32
17. Were sample activities/lesson plans/ tasks/problems provided?	13	87	47	53
18. Was the specific use of calculators indicated?	25	75	32	68
19. Was the specific use of computers indicated?	14	86	21	79
20. Did the teaching strategies include the use of mathematics equipment?	16	84	42	58

Evaluation Practices	School % Yes	School % No	Board % Yes	Board % No
21. Were theoretical or background statements about assessment/evaluation included?	21	79	26	74
22. Were evaluation practices described?	17	83	11	89
23. Were the following evaluation procedures included?				
• Formative evaluation				
— records of previous achievement	4	96	5	95
— topic or unit pretests (ranging from informal oral questioning to written diagnostic tests)	12	88	11	89
— systematic observations of attitudes and performance, including oral and written presentations of solutions to problems	60	40	42	58
— informal questioning related to specific objectives	6	94	100	
— teacher-student discussions or interviews, which could include informal or formal diagnostic strategies to probe levels of understanding	7	93	11	89
— self-evaluation by students	6	94	5	95
— peer evaluation by students	3	97	100	
• Summative evaluation				
— topic or unit post-tests	89	11	74	26
— formal examinations at specific intervals	82	18	63	37
— standardized tests related to course objectives	2	98	5	95
— projects or essays	41	59	26	74
24. Was the relative emphasis/weight of evaluation components for student marks/grades outlined?	68	32	53	47
25. Was information on reporting to parents outlined?	6	94	5	95

Resources	School % Yes	School % No	Board % Yes	Board % No
26. Was a textbook or other learning materials indicated?	94	6	68	32
27. Was a list of reference books or other professional materials for use by teachers provided?	43	57	42	58
28. Was the use of concrete manipulative aids outlined?	8	92	100	

Appendix G: Committee Members

This appendix lists members of the following committees: item development, face validity check, marking of screening materials, marking reliability, planning documents analysis, and interpretation panel. All committees had both English-speaking and French-speaking members and were divided into English and French subcommittees to deal with issues specific to each language. General issues that applied to the reviews in both languages were discussed by all members of a given committee.

Ministry of Education personnel were responsible for directing the various activities for the committees and for providing in-service for the board contacts and support for schools and boards involved in the review.

Ministry of Education

George Blackwood, Ph.D.	Learning Assessment Branch
Dana D'Amore	Learning Assessment Branch
Michael Kozlow, Ph.D.	Learning Assessment Branch
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Robert Morrow	Learning Assessment Branch
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Planning Documents Analysis Committee

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